

$$1. (1) \quad (x^2 - 2y^2)dx + y \cdot x dy = 0$$

$$(1 - 2(\frac{y}{x})^2)dx + \frac{y}{x}dy = 0$$

$$\frac{dy}{dx} = y^2 = u^2 x + u = \frac{2u^2 - 1}{u}$$

$$y^2 = u^2 x + u \Leftarrow u = \frac{y}{x} \text{ (NO)}$$

$$\frac{du}{dx} \cdot x = \frac{u^2 - 1}{u}$$

$$y \neq 0 \Leftarrow u \neq 0 \text{ and } \neq 0 \\ \text{then if } u \neq 0 \text{ then } u \neq 0$$

$$\int \frac{u}{u^2 - 1} du = \int \frac{1}{x} dx$$

$$\text{then when } u^2 - 1 = 0 \text{ or } 0 \\ u = \pm 1 \Leftarrow u = (\frac{y}{x})^2 = 1 \Leftarrow \\ u^2 = 0 \Leftarrow \\ u = \frac{2 \cdot (1)^2 - 1}{(1)^2} = 0 \\ \text{then } y = \pm x \text{ is real}$$

$$\frac{\ln|u^2 - 1|}{2} = \ln|x| + C_1$$

$$\ln|u^2 - 1| = \ln|x^2| + C_2$$

$$u^2 - 1 = Cx^2$$

$$\frac{y^2}{x^2} = Cx^2 + 1 \Rightarrow y^2 = Cx^4 + x^2$$

$$y = \pm \sqrt{Cx^4 + x^2}$$

$$\text{ie } y = \pm x$$

$$\boxed{y = \pm \sqrt{Cx^4 + x^2}}$$

$$(2) \quad (x-y)dx + (x-4y)dy = 0$$

$$\frac{dy}{dx} = -\frac{(x-y)}{(x-4y)} = -\frac{(1-\frac{y}{x})}{(1-4\frac{y}{x})}$$

$$u'x + u = -\frac{(1-u)}{(1-4u)}$$

$$y^2 = u^2 x + u \Leftarrow u = \frac{y}{x} \text{ (NO)}$$

$$u'x + u = \left(\frac{u-1}{1-4u}\right) - u = +\frac{u-1-4u+4u^2}{1-4u}$$

$$u'x = \frac{4u^2 - 1}{1-4u} \Rightarrow u^2 = \frac{4u^2 - 1}{1-4u} \cdot \frac{1}{x} \Rightarrow \frac{du}{dx} = \frac{4u^2 - 1}{1-4u} \cdot \frac{1}{x}$$

$$\frac{1-4u}{4u^2-1} du = \frac{1}{x} dx$$

$$\frac{A}{2u-1} + \frac{B}{2u+1} = \frac{1-4u}{(2u-1)(2u+1)} \\ A = -\frac{1}{2}, B = -\frac{3}{2}$$

$$-\frac{1}{2} \int \frac{1}{2u-1} du - \frac{3}{2} \int \frac{1}{2u+1} du = \int \frac{1}{x} dx$$

$$-\frac{1}{4} \ln|2u-1| - \frac{3}{4} \ln|2u+1| = \ln|x| + C$$

$$\ln |[(2u-1)(2u+1)^3]^{-\frac{1}{4}}| = \ln|x| + C$$

$$(2u-1)(2u+1)^3 = Cx^{-4}$$

$$\boxed{(2\frac{y}{x}-1)(2\frac{y}{x}+1)^3 = Cx^{-4}}$$

$$(2) (x+2y)dx + (2x+3y)dy = 0 \quad \begin{array}{l} z = \frac{y}{x} \text{ (no)} \\ y = zx \end{array}$$

$$\frac{dy}{dx} = -\frac{(x+2y)}{2x+3y} = -\frac{1+2\frac{y}{x}}{2+\frac{3y}{x}} = -\frac{1+2z}{2+3z}$$

$$2^2 x + z = -\frac{1+2z}{2+3z} \quad (\text{from } y = zx \Rightarrow z = -\frac{2}{3})$$

$$2^2 x = -\frac{1-2z-(2+3z)z}{2+3z} = -\frac{1-2z-2z-3z^2}{2+3z} = \frac{-3z^2-4z-1}{2+3z}$$

$$-\int \frac{3z+2}{3z^2+4z+1} dz = \int \frac{1}{x} dx$$

$$-\int \frac{3z+2}{(3z+1)(z+1)} dz = \int \frac{1}{x} dx$$

$$-\frac{1}{2} \int \frac{1}{z+1} dz - \frac{3}{2} \int \frac{1}{3z+1} dz = \int \frac{1}{x} dx$$

$$-\frac{1}{2} \ln|z+1| - \frac{3}{2} \ln|3z+1| \cdot \frac{1}{3} = \ln|x| + C_1$$

$$\ln|z+1| + \ln|3z+1| = \ln|x^2| + C_2$$

$$(z+1)(3z+1) = x^{-2}$$

$$\boxed{\left(\frac{y}{x}+1\right)\left(3\frac{y}{x}+1\right) = x^{-2} \quad \text{ie} \quad y = -x \quad \text{or} \quad y = -\frac{1}{3}x}$$

$$\frac{A}{3z+1} + \frac{B}{z+1} = \frac{3z+2}{(3z+1)(z+1)}$$

$$A = \frac{3}{2} \quad B = \frac{1}{2}$$

$$\left| \begin{array}{l} \text{if } y = -x \Leftrightarrow z = -1 \\ \text{if } y = -\frac{1}{3}x \Leftrightarrow z = -\frac{1}{3} \end{array} \right.$$

$$(3) 2x dy - 2y dx = \sqrt{x^2 + 4y^2} dx$$

$$2dy - 2\frac{y}{x}dx = \sqrt{1 + 4\frac{y^2}{x^2}} dx$$

$$2dy = (\sqrt{1+4u^2} + 2u)dx$$

$$u'x + u = \frac{\sqrt{1+4u^2} + 2u}{2}$$

$$u'x = \frac{\sqrt{1+4u^2} + 2u - 2u}{2} = \left(\frac{1+4u}{2}\right)^{\frac{1}{2}}$$

$$\int \frac{2}{\sqrt{1+4u^2}} du = \int \frac{1}{x} dx \quad (du = \frac{1}{2}dk \Leftrightarrow u = \frac{1}{2}k \text{ (no)})$$

$$\int \frac{2 \cdot \frac{1}{2}dk}{\sqrt{1+k^2}} = \int \frac{1}{x} dx$$

$$\arcsinh(k) = \ln|x| + C_1$$

$$2u = \frac{1}{2} (e^{\ln|x| + C_1} - e^{-\ln|x| - C_1}) = \frac{1}{2} [C_2 x - \frac{1}{C_2} x]$$

$$\frac{y}{x} = C_2 x - \frac{1}{C_2 x}$$

$$\boxed{y = C_2 x^2 + C_3}$$

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$$2. \underline{k_1} \quad y' + \frac{2}{x}y = \frac{1}{x^2} \quad (x > 0)$$

$$u(x) = e^{\int \frac{2}{x} dx} = e^{2\ln|x|} = x^2$$

$$y' \cdot x^2 + 2x \cdot y = 1$$

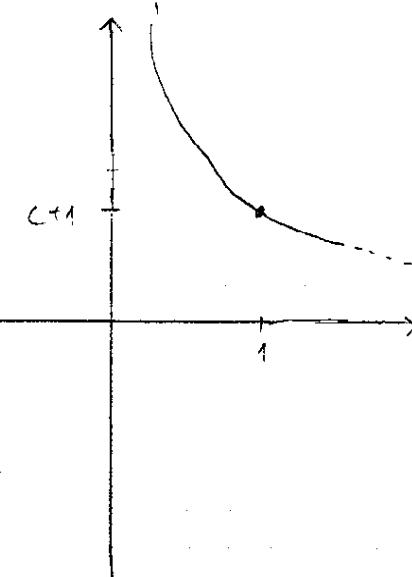
$$\int (y \cdot x^2)' dx = \int 1 dx$$

$$y \cdot x^2 = x + C$$

$$\boxed{y = \frac{1}{x} + \frac{C}{x^2}}$$



(答) 3226) の 答え (3N)



$$\lim_{x \rightarrow 0} \left(\frac{1}{x} + \frac{C}{x^2} \right) = \infty + \infty = \infty$$

$$2. \quad y' - \frac{1}{x}y = x \quad (x > 0)$$

$$u(x) = e^{-\int \frac{1}{x} dx} = e^{\ln x^{-1}} = \frac{1}{x}$$

$$y' \cdot \frac{1}{x} - \frac{1}{x^2} \cdot y = 1$$

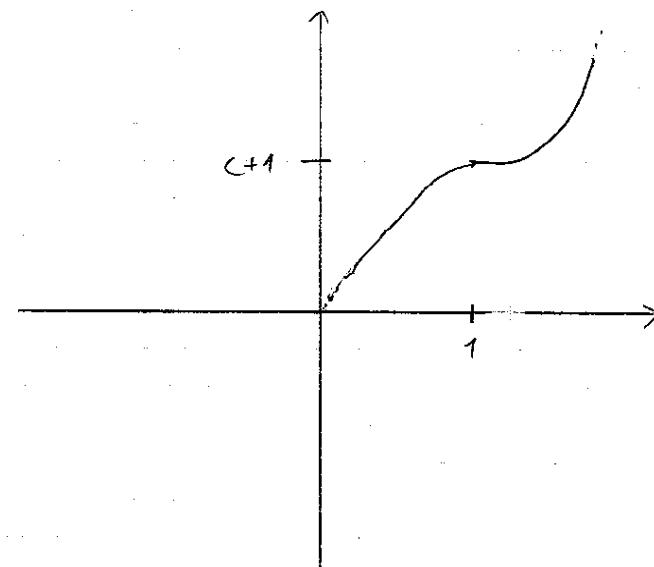
$$\int (y \cdot \frac{1}{x})' dx = \int 1 dx$$

$$y \cdot \frac{1}{x} = x + C$$

$$\boxed{y = x^2 + CX}$$



$$\lim_{x \rightarrow 0^+} x^2 + CX = 0 + 0 = 0$$



3.

$$(k) (x^4 + 2y)dx - xdy = 0$$

$$(x^4 + 2y)dx = xdy$$

$$\frac{dy}{dx} = \frac{x^4 + 2y}{x} = x^3 + \frac{2}{x}y$$

$$y' - \frac{2}{x}y = x^3$$

$$u(x) = e^{\int \frac{2}{x} dx} = e^{2\ln|x|} = x^2$$

; Ejerc. nº 52 (3N)

$$y \cdot x^{-2} - 2x^3y = x$$

$$\int (y \cdot x^{-2})^3 = \int x^3 dx$$

$$y \cdot x^{-2} = \frac{x^2}{2} + C$$

$$\boxed{y = \frac{x^4}{2} + Cx^2}$$



$$(p) (3yx + 3y - 4)dx + (x+1)^2 dy = 0$$

$$(3yx + 3y - 4) = -(x+1)^2 dy$$

$$\frac{dy}{dx} = -\frac{3yx + 3y - 4}{(x+1)^2} = -\frac{3y(x+1) - 4}{(x+1)^2} = -\frac{3y}{(x+1)} + \frac{4}{(x+1)^2}$$

$$y' + \frac{3}{x+1}y = \frac{4}{(x+1)^2}$$

$$u(x) = e^{\int \frac{3}{x+1} dx} = e^{3\ln|x+1|} = (x+1)^3$$

; Ejerc. nº 52 (3N)

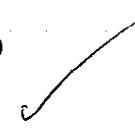
$$(x+1)^3 \cdot y' + 3(x+1)^2 y = 4(x+1)$$

$$\int ((x+1)^3 \cdot y) dx = 4 \int x dx + \int 4 dx$$

$$(x+1)^3 y = 2x^2 + 4x + C_1 - 2 + 2$$

$$(x+1)^3 y = 2(x+1)^2 + (C_1 - 2)$$

$$\boxed{y = \frac{2}{(x+1)} + \frac{C}{(x+1)^3}}$$



$$3. (c) \quad x(x^2+1)y' + 2y = (x^2+1)^2$$

$$\begin{aligned} y' + \frac{2}{x(x^2+1)}y &= \frac{(x^2+1)^2}{x} \\ u(x) &= e^{2\int \frac{1}{x(x^2+1)}dx} = e^{2\int \frac{1}{x}dx - \int \frac{2x}{x^2+1}dx} \\ &= e^{\ln|x^2| - \ln|x^2+1|} = \frac{x^2}{x^2+1} \end{aligned}$$

नियोजित करें (3N)

$$y' \cdot \frac{x^2}{x^2+1} + \frac{2x}{(x^2+1)^2}y = (x^2+1)x$$

$$\int (y \cdot \frac{x^2}{x^2+1})' dx = \int x^3 dx + \int x dx$$

$$y \cdot \frac{x^2}{x^2+1} = \frac{x^4}{4} + \frac{x^2}{2} + C = \frac{x^4+2x^2}{4} + C$$

$$y = \frac{x(x^2+2)(x^2+1)}{4x^2} + C \cdot \frac{x^2+1}{x^2}$$

$$\boxed{y = \frac{(x^2+2)(x^2+1)}{4} + C \cdot \frac{x^2+1}{x^2}}$$

$$4. \quad y' + ax = f(x)$$

$$u(x) = e^{\int a dx} = e^{ax}$$

नियोजित करें (3N)

$$y' \cdot e^{ax} + a \cdot e^{ax}y = f(x) \cdot e^{ax}$$

$$\int (y \cdot e^{ax})' dx = \int f(x) \cdot e^{ax} dx$$

परं |f(x)| \leq M \quad \text{D. 1N}

$$\textcircled{1} \quad \int (y \cdot e^{ax})' dx \leq M \cdot \int e^{ax} dx = M \cdot \frac{e^{ax}}{a} + CM$$

$$y \cdot e^{ax} \leq M \cdot \frac{e^{ax}}{a} + CM$$

$$y \leq \frac{M}{a} + \frac{CM}{e^{ax}} \xrightarrow[x \rightarrow \infty]{} \frac{M}{a}$$

$$\textcircled{2} \quad \int (y \cdot e^{ax})' dx \geq -M \int e^{ax} dx = -M \frac{e^{ax}}{a} - CM$$

$$y \cdot e^{ax} \geq -M \frac{e^{ax}}{a} - CM$$

$$y \geq -\frac{M}{a} - \frac{CM}{e^{ax}} \xrightarrow[x \rightarrow \infty]{} -\frac{M}{a}$$

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5.

$$(c) \quad y' - \frac{1}{2x}y = 5x^2y^5$$

$$y^5 \cdot y' - \frac{1}{2x}y^4 = 5x^2$$

$$\frac{z'}{-4} - \frac{1}{2x}z = 5x^2 \quad | \text{ LGS } \quad z' = -4y^5 \cdot y' \Leftrightarrow z = y^{-4} \quad (\text{no})$$

$$z' + \frac{2}{x}z = -20x^2$$

$$u(x) = e^{\int \frac{2}{x} dx} = e^{2 \ln|x|} = e^{\ln|x|^2} = x^2 \quad (\text{Okt. auf der re. Seite})$$

$$z' \cdot x^2 + 2x z = -20x^4$$

$$\int (z \cdot x^2)' dx = -20 \int x^4 dx$$

$$z \cdot x^2 = -20 \cdot \frac{x^5}{5} + C = -4x^5 + C$$

$$z = -4x^3 + Cx^{-2}$$

$$y^{-4} = -4x^3 + Cx^{-2}$$

$$y = \pm (-4x^3 + Cx^{-2})^{-\frac{1}{4}} \quad \text{ac} \quad y \neq 0 \quad \checkmark$$

$$(2) \quad (x+1)(y \cdot y' - 1) = y^2$$

$$(xy + y)y' - y^2 = x + 1$$

$$y' - \frac{1}{x+1}y = \frac{x+1}{x+1-y} = \frac{1}{y} = y^{-1}$$

$$y \cdot y' - \frac{1}{x+1}y^2 = 1$$

$$\frac{1}{2}z' - \frac{1}{x+1}z = 1 \quad | \text{ O } \quad z' = 2y \cdot y' \Leftrightarrow z = y^2 \quad (\text{no})$$

$$z' - \frac{2}{x+1}z = 2$$

$$u(x) = e^{-2 \int \frac{1}{x+1} dx} = e^{-2 \ln|x+1|^2} = (x+1)^{-2}$$

$$(x+1)^{-2} \cdot z' - \frac{2}{(x+1)^3} \cdot z = 2(x+1)^{-2}$$

$$\int (z \cdot (x+1)^{-2})' dx = 2 \cdot \int (x+1)^{-2} dx$$

$$z \cdot (x+1)^{-2} = -2 \cdot (x+1)^{-1} + C$$

$$z = -2(x+1) + C(x+1)^2$$

$$y^2 = -2(x+1) + C(x+1)^2$$

$$5. (c) \quad x = (x^2 - 2x + 1)y$$

$$x = (x^2 - 2x + 1) \frac{dy}{dx}$$

$$\frac{dx}{dy} x = (x^2 - 2x + 1)$$

$$z \cdot z = (z^2 - 2z + 1)$$

$$: y = k, x = z \text{ (no)}$$

$$z \cdot z - z^2 = -2z + 1$$

$$w^2 = 2z \cdot z \Leftarrow w = z^2 \text{ (no)}$$

$$\frac{w^2}{2} - w = -2z + 1$$

$$w^2 - 2w = -4z + 2$$

$$u(x) = e^{-2x} = e^{-2k}$$

mitteilen soll (3N)

$$\int (w \cdot e^{-2k})' dk = \int (2e^{-2k} - 4k \cdot e^{-2k}) dk$$

$$w \cdot e^{-2k} = 2k \cdot e^{-2k} + C$$

$$w = 2k + C \cdot e^{2k}$$

$$\boxed{x^2 = 2y + C \cdot e^{2y}}$$



$$(3) \quad x^2 y' + 2xy = y^3$$

$$y' + \frac{2}{x} y = \frac{1}{x^2} \cdot y^3$$

$$y^{-3} y' + \frac{2}{x} y^{-2} = \frac{1}{x^2}$$

$$\text{pfi } z^3 = -2y^3 \cdot y \Leftarrow z = y^2 \text{ (no)}$$

$$\frac{z^3}{2} + \frac{2}{x} \cdot z = \frac{1}{x^2}$$

$$z^3 - \frac{4}{x} z = \frac{-2}{x^2}$$

$$u(x) = e^{-4 \int \frac{1}{x} dx} = e^{-4 \ln |x|} = x^{-4}$$

mitteilen soll (3N)

$$x^{-4} \cdot z - 4x^{-5} \cdot z = -2x^{-6}$$

$$\int (x^{-4} \cdot z)' dx = -2 \int x^{-6} dx$$

$$x^{-4} z = -2 \cdot \frac{x^{-5}}{5} + C = \frac{2}{5} x^{-5} + C$$

$$z = \frac{2}{5x} + Cx^4 \Rightarrow y^{-2} = \frac{2}{5x} + Cx^4$$

$$\boxed{y = \pm \left(\sqrt{\frac{2}{5x} + Cx^4} \right)^{-1} \text{ für } y \neq 0}$$



7. $\frac{dy}{dx} = \frac{y}{x}$ ו- $y(0) = 1$ (ב- $x=0$ הערך של y לא מוגדר) \Rightarrow $y(x) = Cx$

הוכחה: נוכיח כי $y(x) = Cx$ היא פתרון.

נוכיח כי $y(x) = Cx$ היא פתרון.

$$[x - (x - y \frac{dy}{dx})] = 0 \Leftrightarrow (x - x + y \frac{dy}{dx}) = 0 \Leftrightarrow y \frac{dy}{dx} = 0$$

לפיכך $y(x) = Cx$ היא פתרון.

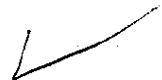
$$\underline{y \cdot \left(x - x + y \frac{dy}{dx} \right)} = a^2$$

$$\frac{1}{2} y^2 \cdot \frac{dy}{dx} = a^2$$

$$\frac{dy}{dx} = \frac{1}{2a^2} y^2$$

$$\int \frac{dy}{y^2} = \int \frac{dx}{2a^2}$$

$$-\frac{1}{y} = \frac{x}{2a^2} + C_1$$



$$-\frac{1}{y} = \frac{x+C}{2a^2}$$

$$\boxed{y = -\frac{2a^2}{x+C}}$$