

$$1. (e) t^2 x'' + 4t x' + 2x = 0 \quad (t > 0)$$

$$x''(t) = r(r-1)t^{r-2} \Leftarrow x'(t) = r t^{r-1} \Leftarrow x(t) = t^r \quad \text{mehr Wörter einfügen}$$

$$r(r-1)t^r - 4t^r \cdot r + 2t^r = 0 \quad \because t^r \neq 0$$

$$r(r-1) - 4r + 2 = 0$$

$$r^2 - r + 4r - 2 = 0 \Rightarrow r^2 + 3r + 2 = 0 \Rightarrow r_{1,2} = \frac{-3 \pm \sqrt{9-8}}{2} = \frac{-3 \pm \sqrt{1}}{2} \quad \begin{matrix} r_1 = -1 \\ r_2 = -2 \end{matrix}$$

$$\boxed{x(t) = c_1 t^{-1} + c_2 t^{-2}}$$



: mehrere Möglichkeiten

$$(f) (t-2)^2 x'' + 3(t-2)x' + x = 0 \quad (t > 2)$$

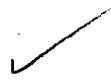
$$x''(t) = r(r-1)(t-2)^{r-2} \Leftarrow x'(t) = r(t-2)^{r-1} \Leftarrow x(t) = (t-2)^r \quad \text{mehr Wörter einfügen}$$

$$r(r-1) \cdot (t-2)^r + 3r(t-2)^{r-1} + (t-2)^r = 0 \quad \because (t-2)^r \neq 0$$

$$r(r-1) + 3r + 1 = 0$$

$$r^2 - r + 3r + 1 = 0 \Rightarrow r^2 + 2r + 1 = 0 \Rightarrow (r+1)^2 = 0 \Rightarrow \boxed{r_1 = -1}$$

$$\boxed{x(t) = c_1 \cdot (t-2)^{-1} + c_2 \cdot (t-2)^{-1} \cdot \ln|t-2|}$$



: mehrere Möglichkeiten

$$2. (g) t^2 x'' + 7t x' + 2x = 0 \quad (t > 0)$$

$$r(r-1) \cdot t^r + 7r \cdot t^r + 2t^r = 0 \quad \because t^r \neq 0 \quad ; x(t) = t^r \quad \text{mehr}$$

$$r(r-1) + 7r + 2 = 0 \Rightarrow r^2 - r + 7r + 2 = 0$$

$$r^2 + 6r + 2 = 0 \Rightarrow r_{1,2} = \frac{-6 \pm \sqrt{36-8}}{2} = \frac{-6 \pm \sqrt{28}}{2} = \frac{-6 \pm 2\sqrt{7}}{2} = \underline{\underline{-3 \pm \sqrt{7}}}$$

: mehrere Möglichkeiten

$$\boxed{x(t) = c_1 t^{-3+\sqrt{7}} + c_2 t^{-3-\sqrt{7}}}$$



$$(h) t^2 x'' + 5t x' + 4x = 0 \quad (t > 0)$$

$$r(r-1) t^r + 5r \cdot t^r + 4t^r = 0 \quad \because t^r$$

; x(t) = t^r  $\quad \text{mehr}$

$$r^2 - r + 5r + 4 = 0 \Rightarrow r^2 + 4r + 4 = 0 \Rightarrow r_{1,2} = \frac{-4 \pm \sqrt{16-16}}{2} = \frac{-4 \pm 0}{2} = -2$$

$$\boxed{x(t) = c_1 t^{-2} + c_2 t^{-2} \cdot \ln|t|}$$



$$2. (2) t^2 \cdot x'' + t \cdot x' + x = 0 \quad (t > 0)$$

$$r(r-1) \cdot t^r + r \cdot t^r + t^r = 0 \quad \because t^r \neq 0$$

$x(t) = t^r$  מושג פונקציונלי

$$r^2 - r + r + 1 = 0$$

$$r^2 + 1 = 0$$

$$v_{1,2} = \pm i$$

$$x(t) = c_1 t^0 \cdot \cos(1 \cdot \ln t) + c_2 t^0 \cdot \sin(1 \cdot \ln t)$$

$$\boxed{x(t) = c_1 \cos(\ln t) + c_2 \sin(\ln t)}$$

טב נסחף יפה

$$(3) t^2 \cdot x'' + 3t \cdot x' + x = 0 \quad (t > 0)$$

$$r(r-1) \cdot t^r + 3r \cdot t^r + t^r = 0 \quad \because t^r \neq 0$$

$x(t) = t^r$  מושג פונקציונלי

$$r^2 - r + 3r + 1 = 0$$

$$r^2 + 2r + 1 = 0$$

$$(r+1)^2 = 0$$

$$v_{1,2} = -1$$

$$\boxed{x(t) = c_1 \cdot t^{-1} + c_2 \cdot t^{-1} \cdot \ln|t|}$$

טב נסחף יפה

$$3. (e) \quad x^2 y'' + xy' = x \ln x$$

$$y(x) = x^r \quad \text{הנורמלית שפונקציית העדיף}$$

$$r(r-1)x^r + rx^r = 0$$

$$r^2 - r + r = 0 \Rightarrow r^2 = 0 \Rightarrow r_{1,2} = 0$$

$$y(x) = c_1 x^0 + c_2 x^0 \ln|x|$$

בגדי פונקציית העדיף

$$\boxed{y_h(x) = c_1 + c_2 \ln|x|}$$

לפונקציית העדיף נסובב

$g(z) = e^z \cdot z$  ( $\ln z = \ln x$  NO),  $g(x) = x \ln x$  הערך של  $g(x)$  נקבע על ידי הערך של  $g(z)$  כפולה, מינימום ומקסימום.

$$y_p(z) = x^0 \cdot e^{z-1} \cdot Q_1(z) = x^0 \cdot e^z \cdot (az+b)$$

ריבוי

$$y_p(x) = x^1 \cdot (a \ln|x| + b) = ax \ln|x| + bx$$

$$y_p'(x) = a \ln|x| + ax \cdot \frac{1}{x} + b = a \ln|x| + (a+b)$$

$$y_p''(x) = \frac{a}{x}$$

$$x^2 \cdot \frac{a}{x} + x \cdot (a \ln|x| + (a+b)) = x \ln|x| \quad |: x \neq 0$$

בנוסף לערך

$$a + a \ln|x| + a+b = \ln|x|$$

$$\left\{ \begin{array}{l} a=1 \\ \end{array} \right.$$

$$2a+b = 0 \Rightarrow 2+b = 0 \Rightarrow \boxed{b=-2}$$

$$\boxed{y_p(x) = x \ln|x| - 2x}$$

$$y(x) = c_1 + c_2 \ln|x| + x \ln|x| - 2x$$



בגדי פונקציית העדיף

$$3. (o) x^2 y'' - 2y = x^2 - \frac{1}{x}$$

נקה ל- $y(x)$  מ- $x^r$  ו- $\ln|x|$  (בנוסף ל- $1/x$ ) (בנוסף ל- $1/x$ ) (בנוסף ל- $1/x$ )

$$r(r-1)x^r - 2x^r = 0 \quad | : x^r \neq 0$$

$$r^2 - r - 2 = 0$$

$$r_{1,2} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} \quad \begin{cases} r_1 = -1 \\ r_2 = 2 \end{cases}$$

$$y(x) = C_1 \cdot x^{-1} + C_2 \cdot x^2$$

נקה ל- $y(x)$  מ- $x^{-1}$  ו- $x^2$

$$(1) \quad x^2 \cdot y'' - 2y = x^2$$

$$(2) \quad x^2 \cdot y'' - 2y = -\frac{1}{x}$$

$$g(x) = x^2 \Rightarrow g(z) = e^{2z} \quad \text{לעומת } y(x) \text{ מ-} x^2 \text{ (בנוסף ל-} z \text{)} \quad (1) \quad \text{נקה}$$

$$g(x) = -x^{-1} \Rightarrow g(z) = -e^{-z} \quad \text{לעומת } y(x) \text{ מ-} -x^{-1} \text{ (בנוסף ל-} z \text{)} \quad (2) \quad \text{נקה}$$

$$Y_{p_1}(z) = z^1 \cdot e^{2z} \cdot Q_0(x) = a_1 z \cdot e^{2z} \Rightarrow Y_{p_1}(x) = a_1 \cdot \ln|x| \cdot x^2 \quad (1)$$

$$Y_{p_2}(z) = z^1 \cdot e^{-z} \cdot Q_0(x) = a_2 z \cdot e^{-z} \Rightarrow Y_{p_2}(x) = a_2 \cdot \ln|x| \cdot \frac{1}{x}$$

$$Y_{p_1}(x) = a_1 \cdot \frac{1}{x} \cdot x^2 + a_1 \cdot \ln|x| \cdot 2x = a_1 x + 2a_1 \cdot \ln|x| \cdot x$$

$$Y_{p_2}(x) = a_2 + 2a_2 \cdot \ln|x| + 2a_2 \cdot x \cdot \frac{1}{x} = 3a_2 + 2a_2 \cdot \ln|x|$$

$$Y_p(x) = a_2 \cdot \frac{1}{x^2} - a_2 \ln|x| \cdot \frac{1}{x^2} = \frac{a_2}{x^2} (1 - \ln|x|)$$

$$Y_p''(x) = a_2 \cdot (-2) \cdot \frac{1}{x^3} (1 - \ln|x|) - \frac{a_2}{x^2} \cdot (-\frac{1}{x}) = \frac{a_2}{x^3} (2 \ln|x| - 3)$$

$$(1) \quad x^2 \cdot (3a_1 + 2a_1 \ln|x|) - 2(a_1 \ln|x| \cdot x^2) = x^2 \quad | : x^2 \neq 0$$

$$3a_1 + 2a_1 \ln|x| - 2a_1 \ln|x| = 1 \Rightarrow \boxed{a_1 = \frac{1}{3}}$$

$$(2) \quad x^2 \left[ \frac{a_2}{x^3} (2 \ln|x| - 3) \right] - 2a_2 \cdot \ln|x| \cdot \frac{1}{x} = -\frac{1}{x} \quad | : x \neq 0$$

$$2a_2 \ln|x| - 3a_2 - 2a_2 \ln|x| = -1 \Rightarrow \boxed{a_2 = \frac{1}{3}}$$

$$Y_p(x) = \frac{1}{3} \ln|x| \cdot x^2 + \frac{1}{3} \ln|x| \cdot \frac{1}{x} \quad \text{נקה ל-} Y_p(x) \text{ מ-} \frac{1}{x}$$

נקה ל- $Y(x)$

$$Y(x) = C_1 \cdot x^{-1} + C_2 \cdot x^2 + \frac{1}{3} \ln|x| \cdot x^2 + \frac{1}{3} \ln|x| \cdot \frac{1}{x}$$

$$4.4) \quad t^3 \cdot x''' + t^2 \cdot x'' - 2t \cdot x' + 2x = t^3 \sin t \quad (t > 0)$$

$$x(t) = t^r \quad \text{ר'גון פ'גון (בונן)} \quad \text{לפ'גון נ'גון}$$

$$r(r-1)(r-2) t^r + r(r-1)t^r - 2rt^r + 2t^r = 0 \quad \forall t^r \neq 0$$

$$r(r-1)(r-2) + r(r-1) - 2(r-1) = 0$$

$$(r-1)[r^2 - 2r + r - 2] = 0$$

$$(r-1)[r^2 - r - 2] = 0$$

$$r_1 = 1 \quad r_{2,3} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} \quad \begin{cases} r_2 = -1 \\ r_3 = 2 \end{cases}$$

$$\boxed{x_h(t) = c_1 t + c_2 t^{-1} + c_3 t^2}$$

לפ'גון י'גון, 1/8

ר'גון פ'גון, 1/6 נ'גון (בונן)

$$x_p(t) = u_1 x_1 + u_2 x_2 + u_3 x_3$$

$$x_p'(t) = u_1' x_1 + u_2' x_2 + u_3' x_3$$

$$x_p'(t) = u_1 x_1 + u_2 x_2 + u_3 x_3 \quad (p)1 \quad u_1' x_1 + u_2' x_2 + u_3' x_3 = 0 \quad (p)2$$

$$x_p''(t) = u_1' x_1 + u_2' x_2 + u_3' x_3$$

$$x_p''(t) = u_1 x_1 + u_2 x_2 + u_3 x_3 \quad (p)3 \quad u_1' x_1 + u_2' x_2 + u_3' x_3 = 0 \quad (p)4$$

$$x_p'''(t) = u_1' x_1''' + u_2' x_2''' + u_3' x_3'''$$

$$f^3(u_1' x_1''' + u_2' x_2''' + u_3' x_3''' + u_1 x_1''' + u_2 x_2''' + u_3 x_3''' ) + t^2 (u_1 x_1'' + u_2 x_2'' + u_3 x_3'')$$

$$- 2t(u_1 x_1' + u_2 x_2' + u_3 x_3') + 2(u_1 x_1 + u_2 x_2 + u_3 x_3) = t^3 \sin t$$

$$u_1(t^3 x_1''' + t^2 x_2'' - 2x_1' + 2x_1) + u_2(t^3 x_2''' + t^2 x_2'' - 2t x_2' + 2x_2)$$

$$+ u_3(t^3 x_3''' + t^2 x_3'' - 2t x_3' + 2x_3) + t^3(u_1 x_1''' + u_2 x_2''' + u_3 x_3''' ) = t^3 \sin t \quad \forall t^3 \neq 0$$

$$\left\{ \begin{array}{l} u_1' x_1 + u_2' x_2 + u_3' x_3 = 0 \\ u_1 x_1' + u_2 x_2' + u_3 x_3' = 0 \\ u_1 x_1''' + u_2 x_2''' + u_3 x_3''' = \sin t \end{array} \right.$$

$$\left\{ \begin{array}{l} u_1' x_1 + u_2' x_2 + u_3' x_3 = 0 \\ u_1 x_1' + u_2 x_2' + u_3 x_3' = 0 \\ u_1 x_1''' + u_2 x_2''' + u_3 x_3''' = \sin t \end{array} \right.$$

$$\left\{ \begin{array}{l} u_1' x_1 + u_2' x_2 + u_3' x_3 = 0 \\ u_1 x_1' + u_2 x_2' + u_3 x_3' = 0 \\ u_1 x_1''' + u_2 x_2''' + u_3 x_3''' = \sin t \end{array} \right.$$

$$\Delta = \begin{vmatrix} t & t^{-1} & t^2 \\ 1 & -t^{-2} & 2t \\ 0 & 2t^{-3} & 2 \end{vmatrix} = t \cdot \begin{vmatrix} -t^{-2} & 2t \\ 2t^{-3} & 2 \end{vmatrix} - 1 \cdot \begin{vmatrix} t^{-1} & t^2 \\ 2t^{-3} & 2 \end{vmatrix}$$

$$= t [-2t^{-2} - 4t^{-2}] - (2t^{-1} - 2t^{-1}) = \underline{\underline{-6t^{-1}}}$$

$$U_1 = \frac{\begin{vmatrix} 0 & t^{-1} & t^2 \\ 0 & -t^{-2} & 2t \\ \sin t & 2t^{-3} & 2 \end{vmatrix}}{-6t^{-1}} = \frac{1}{-6t^{-1}} \sin t \cdot \begin{vmatrix} t^{-1} & t^2 \\ -t^{-2} & 2t \end{vmatrix} \quad \checkmark$$

$$= -\frac{t}{6} \cdot \sin t (2+1) = -\frac{1}{2} \underline{\underline{t \sin t}}$$

$$U_1 = -\frac{1}{2} \oint t \sin t dt = \boxed{\begin{array}{l} u=t \quad v=-\cos t \\ u'=1 \quad v'=\sin t \end{array}} = -\frac{1}{2} (-t \cos t + \cancel{\frac{1}{2} \cos t}) \Rightarrow \boxed{U_1 = \frac{1}{2} t \cos t - \frac{1}{2} \sin t}$$

$$U_2 = \frac{\begin{vmatrix} t & 0 & t^2 \\ 1 & 0 & 2t \\ 0 & \sin t & 2 \end{vmatrix}}{-6t^{-1}} = -\frac{t}{6} \cdot (-\sin t) \cdot \begin{vmatrix} t & t^2 \\ 1 & 2t \end{vmatrix} = \frac{t}{6} \sin t (2t^2 - t^2) = \frac{t^3}{6} \sin t$$

$$U_2 = \frac{1}{6} \oint t^3 \sin t dt = \boxed{\begin{array}{l} u=t^3 \quad v=-\cos t \\ u'=3t^2 \quad v'=\sin t \end{array}} = \frac{1}{6} [-t^3 \cos t + 3 \cancel{\int t^2 \cos t dt}] =$$

$$= \boxed{\begin{array}{l} u=t^2 \quad v=\sin t \\ u'=2t \quad v'=\cos t \end{array}} = \frac{1}{6} [-t^3 \cos t + 3(t^2 \sin t - 2 \cancel{\int t \sin t dt})] =$$

$$= \frac{1}{6} \left\{ -t^3 \cos t + 3[t^2 \sin t - 2(-t \cos t + \sin t)] \right\} =$$

$$= -\frac{1}{6} t^3 \cos t + \frac{1}{2} [t^2 \sin t - 2(-t \cos t + \sin t)] = -\frac{1}{6} t^3 \cos t + \frac{1}{2} t^2 \sin t + t \cos t - \sin t$$

$$U_3 = \begin{vmatrix} t & t^{-1} & 0 \\ 1 & -t^{-2} & 0 \\ 0 & 2t^{-3} & \sin t \end{vmatrix} \cdot \frac{t}{-6} = \frac{t}{-6} \cdot \sin t \begin{vmatrix} t & t^{-1} \\ 1 & -t^{-2} \end{vmatrix} = \frac{t}{-6} \cdot \sin t (-t^{-1} - t^{-1}) = \frac{1}{3} \sin t$$

$$U_3 = \frac{1}{3} \sin t dt = \frac{1}{3} (-\cos t) = \underline{\underline{-\frac{1}{3} \cos t}}$$

$$Y_p(t) = t \cdot (\frac{1}{2} t \cos t - \frac{1}{2} \sin t) + t^{-1} \left[ -\frac{1}{6} t^3 \cos t + \frac{1}{2} t^2 \sin t + t \cos t - \sin t \right] + t^2 \cdot (-\frac{1}{3} \cos t)$$

$$= \frac{1}{2} t^2 \cos t - \frac{1}{2} t \sin t - \frac{1}{6} t^3 \cos t + \frac{1}{2} t^2 \sin t + \cos t - t^{-1} \sin t - \frac{1}{3} t^2 \cos t$$

$$= \cos t - t^{-1} \sin t.$$

$$4. (P) \quad t^3 \cdot x''' - 3t^2 x'' + 6tx' - 6x = t^4 \ln t$$

$$\therefore x(t) = t^r \quad \text{נמצא רמה בונן גורנין רמה שלישית}$$

$$r(r-1)(r-2)t^r - 3r(r-1)t^r + 6rt^r - 6t^r = 0$$

$$t^r [ (r-1)[r(r-2)-3r+6] ] = 0$$

$$(r-1)(r^2 - 2r - 3r + 6) = 0$$

$$(r-1)(r^2 - 5r + 6) = 0$$

$$r_1 = 1 \quad r_{2,3} = \frac{s \pm \sqrt{2s-24}}{2} = \frac{5 \pm 1}{2} < r_2 = 2 < r_3 = 3$$

$$x_h(t) = c_1 t + c_2 t^2 + c_3 t^3$$

השאלה הינה:

$$x(t) = u_1 x_1 + u_2 x_2 + u_3 x_3$$

נמצא רמה שנייה שלישית

$$x'(t) = u_1' x_1 + u_1 x_1' + u_2' x_2 + u_2 x_2' + u_3' x_3 + u_3 x_3'$$

$$x'(t) = u_1 x_1' + u_2 x_2' + u_3 x_3'$$

בנ"ה  $u_1' x_1 + u_2' x_2 + u_3' x_3 = 0$  (1)

$$x''(t) = u_1' x_1' + u_1 x_1'' + u_2' x_2' + u_2 x_2'' + u_3' x_3' + u_3 x_3''$$

$$x''(t) = u_1 x_1'' + u_2 x_2'' + u_3 x_3''$$

בנ"ה  $u_1 x_1'' + u_2 x_2'' + u_3 x_3'' = 0$  (2)

$$x'''(t) = u_1 x_1''' + u_1 x_1''' + u_2 x_2''' + u_2 x_2''' + u_3 x_3''' + u_3 x_3'''$$

$$t^3 (u_1 x_1''' + u_2 x_2''' + u_3 x_3''' ) - 3t^3 (u_1 x_1'' + u_2 x_2'' + u_3 x_3'')$$

$$+ 6t (u_1 x_1' + u_2 x_2' + u_3 x_3') - 6 (u_1 x_1 + u_2 x_2 + u_3 x_3) = t^4 \ln t$$

$$t^3 (u_1 x_1''' + u_2 x_2''' + u_3 x_3''' ) = t^4 \ln t$$

$$\left\{ \begin{array}{l} u_1 x_1 + u_2 x_2 + u_3 x_3 = 0 \\ u_1 x_1' + u_2 x_2' + u_3 x_3' = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} u_1 x_1'' + u_2 x_2'' + u_3 x_3'' = t \ln t \\ u_1 x_1''' + u_2 x_2''' + u_3 x_3''' = t^4 \ln t \end{array} \right.$$

$$\Delta = \begin{vmatrix} t & t^2 & t^3 \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix} = t \begin{vmatrix} 1 & 3t^2 \\ 2 & 6t \end{vmatrix} - 1 \cdot \begin{vmatrix} t^2 & t^3 \\ 2 & 6t \end{vmatrix} \quad \text{: } \Delta \propto t \text{ (2)}$$

$$u_1 = \begin{vmatrix} 0 & t^2 & t^3 \\ 0 & 2t & 3t^2 \\ t\ln t & 2 & 6t \end{vmatrix} \cdot \frac{1}{2t^3} =$$

$$= t\ln t \cdot \begin{vmatrix} t^2 & t^3 \\ 2t & 3t^2 \end{vmatrix} \cdot \frac{1}{2t^3} = \ln t \cdot \frac{1}{2t^2} (3t^4 - 2t^4) = \frac{1}{2} \ln t \cdot t^2$$

$$u_1 = \frac{1}{2} \int t^2 \ln t dt = \boxed{\begin{array}{l} u=t^2 \quad v=t\ln t - t \\ u'=2t \quad v'=\ln t \end{array}} = \frac{1}{2} [t^3 \ln t - t^3 - 2 \int t^2 \ln t dt + 2 \int t^2 dt]$$

$$3 \int t^2 \ln t dt = t^3 \ln t - \frac{1}{3} t^3 \Rightarrow \int t^2 \ln t dt = \frac{1}{3} t^3 \ln t - \frac{1}{9} t^3$$

$$\boxed{u_1 = \frac{1}{6} t^3 \ln t - \frac{1}{18} t^3}$$

$$u_2^{(2)} = \begin{vmatrix} t & 0 & t^3 \\ 1 & 0 & 3t^2 \\ 0 & 6\ln t & 6t \end{vmatrix} \cdot \frac{1}{2t^3} = -t\ln t \cdot \begin{vmatrix} t & t^3 \\ 1 & 3t^2 \end{vmatrix} \cdot \frac{1}{2t^3}$$

$$= -t\ln t \cdot \cancel{[3t^3 - t^3]} \cdot \frac{1}{2t^3} = -t\ln t$$

$$-u_2 = -\int t \ln t dt = \boxed{\begin{array}{l} u=t \quad v=t\ln t - t \\ u'=1 \quad v'=\ln t \end{array}} = t^2 \ln t - t^2 - \int t \ln t dt + \int t dt$$

$$\Rightarrow 2 \int t \ln t dt = t^2 \ln t - t^2 + \frac{1}{2} t^2 = t^2 \ln t - \frac{1}{2} t^2$$

$$\Rightarrow \boxed{u_2 = -\frac{1}{2} t^2 \ln t + \frac{1}{4} t^2}$$

$$u_3^{(3)} = \begin{vmatrix} t & t^2 & 0 \\ 1 & 2t & 0 \\ 0 & 2 & t\ln t \end{vmatrix} \cdot \frac{1}{2t^3} = t\ln t \cdot \begin{vmatrix} t & t^2 \\ 1 & 2t \end{vmatrix} \cdot \frac{1}{2t^3} = t\ln t \cdot \cancel{[2t^2 - t^2]} \cdot \frac{1}{2t^3}$$

$$= t^2 \cdot \ln t \cdot \frac{1}{8t^3} \cdot \frac{1}{2} = \frac{1}{16} \ln t \Rightarrow \boxed{u_3 = \frac{1}{2} t \ln t - \frac{1}{2} t}$$

$$x_p(t) = \frac{1}{6} t^4 \ln t - \frac{1}{18} t^4 - \frac{1}{2} t^4 \ln t - \frac{1}{4} t^4 + \frac{1}{2} t^2 \ln t - \frac{1}{2} t^4 \quad \text{"גזר פונק", 108}$$

$$\boxed{x_p(t) = \frac{1}{6} t^4 \ln t - \frac{11}{36} t^4}$$

$$\boxed{x(t) = C_1 t + C_2 t^2 + C_3 t^3 + \frac{1}{6} t^4 \ln t - \frac{11}{36} t^4}$$

$$q_1, (c) \quad x' + x = \sin t \quad (0 \leq t < \pi)$$

$$x'''(t) = r^3 e^{rt}, x''(t) = r^2 e^{rt}, x'(t) = r e^{rt} \leq x(t) = e^{rt} \quad \text{ר'גנ'ן פ'נו ל'נו} \quad \text{ר'גנ'ן פ'נו ל'נו}$$

$$r^3 + r = 0$$

ר'גנ'ן פ'נו ל'נו

$$r(r^2 + 1) = 0$$

$$r_1 = 0 \quad r_{2,3} = \pm i$$

$$X_u(t) = C_1 \cdot e^0 + C_2 e^0 \cdot \cos(t) + C_3 e^0 \sin(t) \quad \text{: יס'נ'ן פ'נו, 108}$$

$$\boxed{X_u(t) = C_1 + C_2 \cos t + C_3 \sin t} \quad \checkmark$$

$$X_p(t) = u_1 x_1 + u_2 x_2 + u_3 x_3 \quad \text{: ג'ג'ן פ'נו ל'נו} \quad \text{: ג'ג'ן פ'נו ל'נו}$$

$$6) u_1 x_1' + u_2 x_2' + u_3 x_3' = 0 \rightarrow u_1' x_1 + u_2' x_2 + u_3' x_3 = 0 \quad \text{ב'ג'ג'}$$

$$X_p'''(t) = u_1' x_1''' + u_1 x_1'' + u_2' x_2''' + u_2 x_2'' + u_3' x_3''' + u_3 x_3''$$

$$u_1' x_1''' + u_1 x_1'' + u_2' x_2'' + u_2 x_2''' + u_3' x_3'' + u_3 x_3''' + u_1 x_1 + u_2 x_2 + u_3 x_3 = \frac{1}{5} \sin t \quad \text{: יס'נ'ן פ'נו, 213}$$

$$\begin{cases} u_1' x_1 + u_2 x_2 + u_3 x_3 = 0 \\ u_1' x_1' + u_2' x_2 + u_3' x_3' = 0 \\ u_1' x_1'' + u_2' x_2'' + u_3' x_3'' = \frac{1}{5} \sin t \end{cases}$$

$$\Delta = \begin{vmatrix} 1 & \cos t & \sin t \\ 0 & -\sin t & \cos t \\ 0 & -\cos t & -\sin t \end{vmatrix} = 1 \cdot \begin{vmatrix} -\sin t & \cos t \\ -\cos t & -\sin t \end{vmatrix} = \sin^2 t + \cos^2 t = 1 \quad \text{: א'ג'ג' פ'נו}$$

$$u_1' = \begin{vmatrix} 0 & \cos t & \sin t \\ 0 & -\sin t & \cos t \\ \frac{1}{5} \sin t & -\cos t & -\sin t \end{vmatrix} \cdot 1 = \frac{1}{\sin t} \cdot \begin{vmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{vmatrix} = (\cos^2 t + \sin^2 t) \frac{1}{\sin t} = \frac{1}{\sin t} \quad \text{: יס'נ'ן פ'נו}$$

$$u_2' = \begin{vmatrix} 1 & 0 & \sin t \\ 0 & 0 & \cos t \\ 0 & \frac{1}{5} \sin t & -\sin t \end{vmatrix} = -1 \cdot \frac{\cos t}{\sin t} \quad u_3' = \begin{vmatrix} 1 & \cos t & 0 \\ 0 & -\sin t & 0 \\ 0 & -\cos t & \frac{1}{5} \sin t \end{vmatrix} = 1 \cdot \left(-\frac{\sin t}{\sin t}\right) = -1$$

$$u_1 = \int \frac{1}{\sin t} dt = \int \frac{-\sin t}{\sin^2 t} dt = \boxed{z = \cos t} = \int -\frac{1}{(1-z)^2} dz = -\frac{1}{2} \int \frac{dz}{1-z} - \frac{1}{2} \int \frac{dz}{1+z} = \frac{1}{2} [\ln|1-z| - \ln|1+z|]$$

$$u_1 = \frac{1}{2} \ln \left| \frac{1-\cos t}{1+\cos t} \right| = \frac{1}{2} \ln \left| \tan \frac{t}{2} \right| = \ln \left| \tan \frac{t}{2} \right|$$

$$u_2 = \int \frac{\cos t}{\sin t} dt \Rightarrow \boxed{u_2 = -\ln|\sin t|} \quad u_3 = -\int dt \Rightarrow \boxed{u_3 = -t}$$

$$X_p(t) = \ln \left| \tan \frac{t}{2} \right| + \cos t \ln |\sin t| - t \sin t$$

: ג'ג'ן פ'נו

$$\begin{aligned}
 5. (k) \quad & (x+2)^2 \sum_{n=3}^{\infty} n a_n (x+2)^{n-4} - \sum_{n=1}^{\infty} n a_n (x+2)^{n-1} = \\
 & = \sum_{n=3}^{\infty} n a_n (x+2)^{n-2} - \sum_{n=1}^{\infty} n a_n (x+2)^{n+1} \\
 & = \sum_{n=1}^{\infty} (n+2) a_{n+2} (x+2)^n - \sum_{n=2}^{\infty} (n-1) a_{n-1} (x+2)^n \\
 & = \sum_{n=1}^{\infty} (n+2) a_{n+2} (x+2)^n - \sum_{n=1}^{\infty} (n-1) a_{n-1} (x+2)^n \\
 & = \sum_{n=1}^{\infty} [(n+2) a_{n+2} - (n-1) a_{n-1}] (x+2)^n \\
 & \quad : \text{S}p\text{J1} \quad x_0 = -2 \quad / \text{no} \\
 & = \sum_{n=1}^{\infty} [(n+2) a_{n+2} - (n-1) a_{n-1}] (x-x_0)^n
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & x \sum_{n=0}^{\infty} (n-5)^2 b_{n+1} (x-3)^{n+3} = (z+3) \sum_{n=0}^{\infty} (n-5)^2 b_{n+1} z^{n+3} \\
 & z = x-3 \\
 & = \sum_{n=0}^{\infty} (n-5)^2 b_{n+1} z^{n+4} + \sum_{n=0}^{\infty} 3(n-5)^2 b_{n+1} z^{n+3} \\
 & = \sum_{n=1}^{\infty} (n-6)^2 b_n z^{n+3} + \sum_{n=1}^{\infty} 3(n-5)^2 b_{n+1} z^{n+3} + 75 b_1 z^3 \\
 & = \sum_{n=1}^{\infty} [(n-6)^2 b_n + 3(n-5)^2 b_{n+1}] z^{n+3} + 75 b_1 z^3 \\
 & = \sum_{n=1}^{\infty} [(n-6)^2 b_n + 3(n-5)^2 b_{n+1}] (x-3)^{n+3} + 75 b_1 (x-3)^3 \\
 & = \sum_{n=4}^{\infty} [(n-9)^2 b_{n-3} + 3(n-8)^2 b_{n-2}] (x-3)^n + 75 b_1 (x-3)^3 \\
 & \quad : \text{S}p\text{J1} \quad x_0 = 3 \quad / \text{no} \\
 & = \sum_{n=4}^{\infty} [(n-9)^2 b_{n-3} + 3(n-8)^2 b_{n-2}] (x-x_0)^n + 75 b_1 (x-3)^3
 \end{aligned}$$

$$6. f(x) = 3x^2 - 8x + 2$$

$$x=2 \Rightarrow 0$$

∴ परा वृत्त (n>3 लिए)  $f^{(n)}(x)$  परावर्तन करने की क्रिया करें।

$$f(x) = f(2) + f'(2)(x-2) + f''(2)(x-2)^2 \cdot \frac{1}{2} + f'''(2)(x-2)^3 \cdot \frac{1}{6} + 0$$

$$\begin{aligned} f(x) &= 3 \cdot 4 - 16 + 2 + [6 \cdot 2^{\frac{5}{2}} - 8](x-2) + 6 \cdot (x-2)^2 \cdot \frac{1}{2} \\ &= -2 + 4x - 8 + 3x^2 - 12x + 12 \end{aligned}$$

$$\boxed{f(x) = 3x^2 - 8x + 2}$$

, 100