

$$y'' + (x-1)y' - y = 0$$

$$y(0) = 1; y'(0) = 0 \quad x_0 = 0$$

(1) 1

נמצאים את הפונקציה

$$y = \sum_{n=0}^{\infty} a_n x^n; \quad y' = \sum_{n=1}^{\infty} n a_n x^{n-1}; \quad y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

נציב את זה:

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + x \cdot \sum_{n=1}^{\infty} n a_n x^{n-1} - \sum_{n=1}^{\infty} n a_n x^{n-1} - \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n + \sum_{n=1}^{\infty} n a_n x^n - \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n - \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n + \sum_{n=0}^{\infty} n a_n x^n - \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n - \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} [(n+2)(n+1) a_{n+2} + n a_n - (n+1) a_{n+1} - a_n] x^n = 0$$

$$\sum_{n=0}^{\infty} [(n+2)(n+1) a_{n+2} - (n+1) a_{n+1} + (n-1) a_n] x^n = 0$$

$$(n+2)(n+1) a_{n+2} - (n+1) a_{n+1} + (n-1) a_n = 0$$

כלומר

$$a_{n+2} = \frac{a_{n+1}}{n+2} - \frac{(n-1)a_n}{(n+2)(n+1)}$$

נמצאים את

$$y(0) = a_0 = 1$$

נמצאים את האיבר הראשון של הפונקציה

$$y'(0) = a_1 = 0$$

$$a_2 = \frac{a_1}{2} - \frac{(-1)a_0}{2 \cdot 1} = \frac{0}{2} - \frac{-1}{2} = \frac{1}{2}$$

נמצאים את

$$a_3 = \frac{a_2}{3} - \frac{0 \cdot a_1}{3 \cdot 2} = \frac{a_2}{3} = \frac{1}{6}$$

$$a_4 = \frac{a_3}{4} - \frac{1 \cdot a_2}{4 \cdot 3} = \frac{a_3}{4} - \frac{a_2}{12} = \frac{1}{24} - \frac{1}{24} = 0$$

$$a_5 = \frac{a_4}{5} - \frac{2 \cdot a_3}{5 \cdot 4} = \frac{0}{5} - \frac{2 \cdot \frac{1}{6}}{20} = -\frac{1}{10} \cdot \frac{1}{6} = -\frac{1}{60}$$

$$a_6 = \frac{a_5}{6} - \frac{3 \cdot a_4}{6 \cdot 5} = -\frac{1}{60} \cdot \frac{1}{6} - \frac{1}{10} \cdot 0 = -\frac{1}{360}$$

$$y(x) = 1 + \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{60}x^5 - \frac{1}{360}x^6 + \dots$$



$$2y'' + (x+1)y' + 3y = 0 \quad y(2) = 0 \quad y'(2) = 1 \quad x_0 = 2 \quad (2) .1$$

$$y = \sum_{n=0}^{\infty} a_n (x-2)^n \quad ; \quad y' = \sum_{n=1}^{\infty} n \cdot a_n \cdot (x-2)^{n-1} \quad y'' = \sum_{n=2}^{\infty} n(n-1) a_n (x-2)^{n-2}$$

הצבה

$$2 \cdot \sum_{n=2}^{\infty} n(n-1) a_n (x-2)^{n-2} + (x+1) \sum_{n=1}^{\infty} n a_n (x-2)^{n-1} + 3 \sum_{n=0}^{\infty} a_n (x-2)^n = 0$$

$$\sum_{n=0}^{\infty} 2(n+2)(n+1) a_{n+2} (x-2)^n + (x-2+3) \sum_{n=1}^{\infty} n a_n (x-2)^{n-1} + 3 \sum_{n=0}^{\infty} a_n (x-2)^n = 0$$

$$\sum_{n=0}^{\infty} 2(n+2)(n+1) a_{n+2} (x-2)^n + \sum_{n=1}^{\infty} n a_n (x-2)^n + \sum_{n=1}^{\infty} 3n a_n (x-2)^{n-1} + 3 \sum_{n=0}^{\infty} a_n (x-2)^n = 0$$

$$\sum_{n=0}^{\infty} 2(n+2)(n+1) a_{n+2} (x-2)^n + \sum_{n=0}^{\infty} n a_n (x-2)^n + \sum_{n=0}^{\infty} 3(n+1) a_{n+1} (x-2)^n + 3 \sum_{n=0}^{\infty} a_n (x-2)^n = 0$$

$$\sum_{n=0}^{\infty} [2(n+2)(n+1) a_{n+2} + n a_n + 3(n+1) a_{n+1} + 3a_n] (x-2)^n = 0$$

$$\sum_{n=0}^{\infty} [2(n+2)(n+1) a_{n+2} + 3(n+1) a_{n+1} + (n+3) a_n] (x-2)^n = 0$$

השווה מקדמים

$$2(n+2)(n+1) a_{n+2} + 3(n+1) a_{n+1} + (n+3) a_n = 0$$

$$2(n+2)(n+1) a_{n+2} = -3(n+1) a_{n+1} - (n+3) a_n$$

$$a_{n+2} = \frac{-3a_{n+1}}{2(n+2)} - \frac{(n+3)}{2(n+2)(n+1)} a_n$$

הצבה

$$y(2) = a_0 = 0 \Rightarrow a_0 = 0$$

$$y'(2) = a_1 = 1 \Rightarrow a_1 = 1$$

$$a_2 = \frac{-3 \cdot a_1}{2 \cdot 2} - \frac{3 \cdot a_0}{2 \cdot 2 \cdot 1} = -\frac{3}{4} \cdot 1 - 3 \cdot \frac{1}{4} \cdot 0 \Rightarrow a_2 = -\frac{3}{4}$$

$$a_3 = \frac{-3a_2}{2 \cdot 3} - \frac{4 \cdot a_1}{2 \cdot 3 \cdot 2} = -\frac{1}{2} \cdot \left(-\frac{3}{4}\right) - \frac{1}{3} = \frac{3}{8} - \frac{1}{3} = \frac{3}{24} - \frac{8}{24} \Rightarrow a_3 = \frac{1}{24}$$

$$a_4 = \frac{-3 \cdot a_3}{2 \cdot 4} - \frac{5 \cdot a_2}{2 \cdot 4 \cdot 3} = -\frac{3}{8} \cdot \frac{1}{24} - \frac{5}{2 \cdot 3 \cdot 4} \cdot \left(-\frac{3}{4}\right) = -\frac{1}{64} + \frac{5}{32} = \frac{10}{64} - \frac{1}{64} \Rightarrow a_4 = \frac{9}{64}$$

$$a_5 = \frac{-3 \cdot a_4}{2 \cdot 5} - \frac{6 \cdot a_3}{2 \cdot 5 \cdot 4} = -\frac{3}{10} \cdot \frac{9}{64} - \frac{3}{20} \cdot \frac{1}{24} = -\frac{27}{640} - \frac{1}{640} \Rightarrow a_5 = -\frac{28}{640}$$

$$y(x) = x - \frac{3}{4}x^2 + \frac{1}{24}x^3 + \frac{9}{64}x^4 - \frac{28}{640}x^5 + \dots$$

$$2y'' + (x+1)y' + 3y = 0 \quad y(2) = 1 \quad y'(2) = 0 \quad x_0 = 2 \quad (3) \cdot 1$$

(אנחנו רוצים למצוא את הפונקציה)

$$a_{n+2} = \frac{-3a_{n+1}}{2(n+2)} - \frac{(n+3)a_n}{2(n+2)(n+1)}$$

$$y(2) = a_0 = 1 \Rightarrow a_0 = 1$$

$$y'(2) = a_1 = 0 \Rightarrow a_1 = 0$$

$$a_2 = \frac{-3 \cdot a_1}{2 \cdot 2} - \frac{3 \cdot a_0}{2 \cdot 2 \cdot 1} = -\frac{3}{4} \cdot 0 - \frac{3}{4} \cdot 1 \Rightarrow a_2 = -\frac{3}{4}$$

$$a_3 = \frac{-3 \cdot a_2}{2 \cdot 3} - \frac{4 \cdot a_1}{2 \cdot 3 \cdot 2} = -\frac{1}{2} \cdot \left(-\frac{3}{4}\right) - \frac{0}{2} \Rightarrow a_3 = \frac{3}{8}$$

$$a_4 = \frac{-3 \cdot a_3}{2 \cdot 4} - \frac{5 \cdot a_2}{2 \cdot 4 \cdot 3} = -\frac{3}{8} \cdot \frac{3}{8} + \frac{5}{2 \cdot 8 \cdot 4} \cdot \frac{3}{4} = -\frac{9}{64} + \frac{5}{32} \Rightarrow a_4 = \frac{1}{64}$$

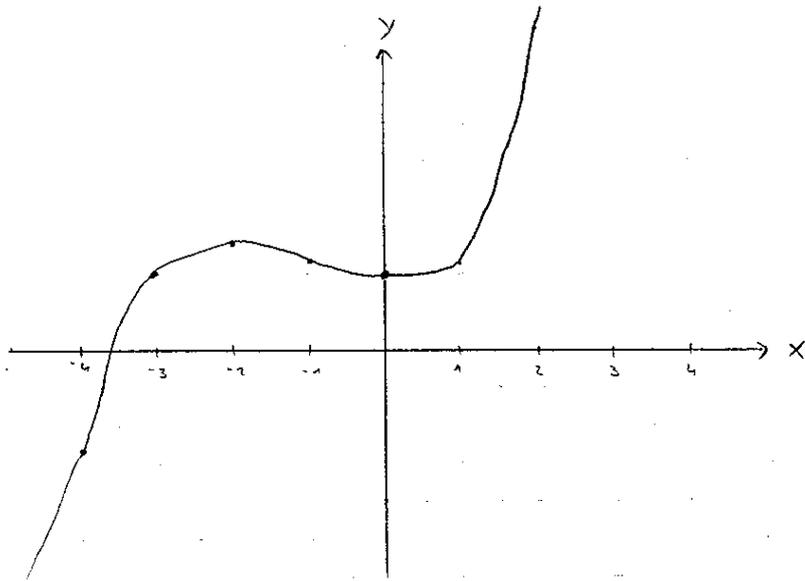
$$a_5 = \frac{-3 \cdot a_4}{2 \cdot 5} - \frac{6 \cdot a_3}{2 \cdot 5 \cdot 4} = -\frac{3}{10} \cdot \frac{1}{64} - \frac{3}{20} \cdot \frac{3}{8} = -\frac{3}{640} - \frac{9}{640} \Rightarrow a_5 = -\frac{39}{640}$$

$$y(x) = 1 - \frac{3}{4}x^2 + \frac{3}{8}x^3 + \frac{1}{64}x^4 - \frac{39}{640}x^5 + \dots$$

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$$Y(x) = 1 + \frac{1}{2}x^2 + \frac{1}{6}x^3$$

הפונקציה נקראת 3 עם קו 1 ונקראת 2



$$Y(0) = 1$$

$$Y(-1) = \frac{4}{3}$$

$$Y(-2) = \frac{5}{3}$$

$$Y(-3) = 1$$

$$Y(-4) = -\frac{5}{3}$$

$$Y(1) = \frac{5}{3}$$

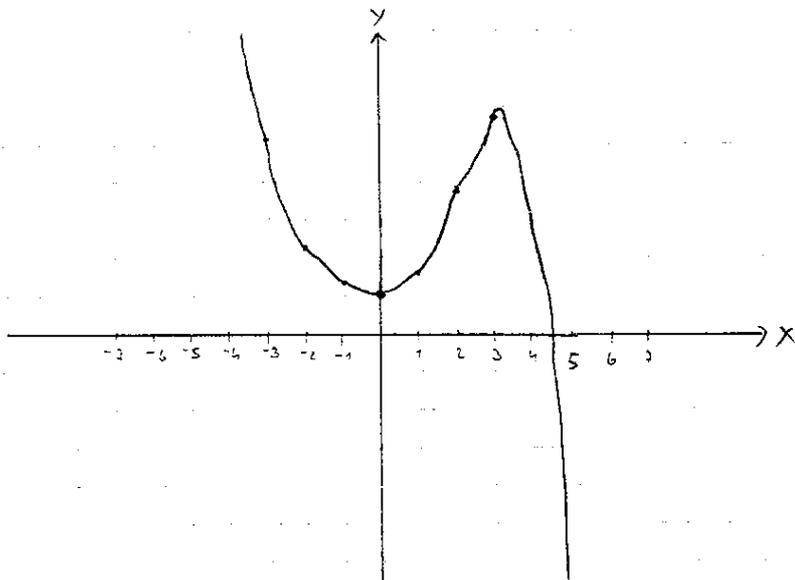
$$Y(2) = \frac{13}{3}$$

$$Y(3) = 10$$



$$Y(x) = 1 + \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{60}x^5$$

הפונקציה נקראת 4 עם קו 1 ונקראת



$$Y(0) = 1$$

$$Y(-1) = 1.35$$

$$Y(-2) = 2.2$$

$$Y(-3) = 5.05$$

$$Y(-4) = 15.4$$

$$Y(1) = 1.65$$

$$Y(2) = 3.8$$

$$Y(3) = 5.95$$

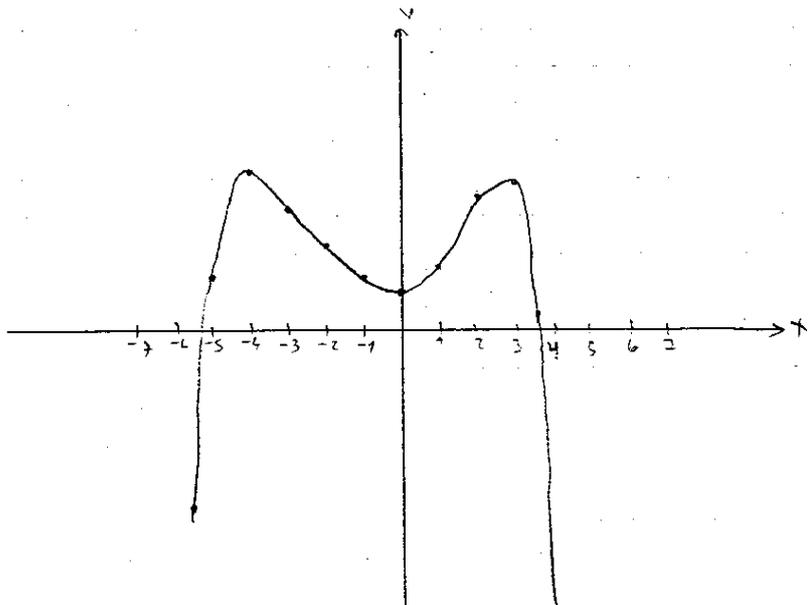
$$Y(4) = 2.6$$

$$Y(5) = -17.75$$



$$Y(x) = 1 + \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{60}x^5 - \frac{1}{360}x^6$$

הפונקציה נקראת 5 עם קו 1 ונקראת



$$Y(0) = 1$$

$$Y(-1) = 1.347$$

$$Y(-2) = 2.02$$

$$Y(-3) = 3.025$$

$$Y(-4) = 4.02$$

$$Y(-5) = 1.3472$$

$$Y(-5.5) = -4.61$$

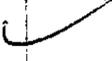
$$Y(1) = 1.64$$

$$Y(2) = 3.62$$

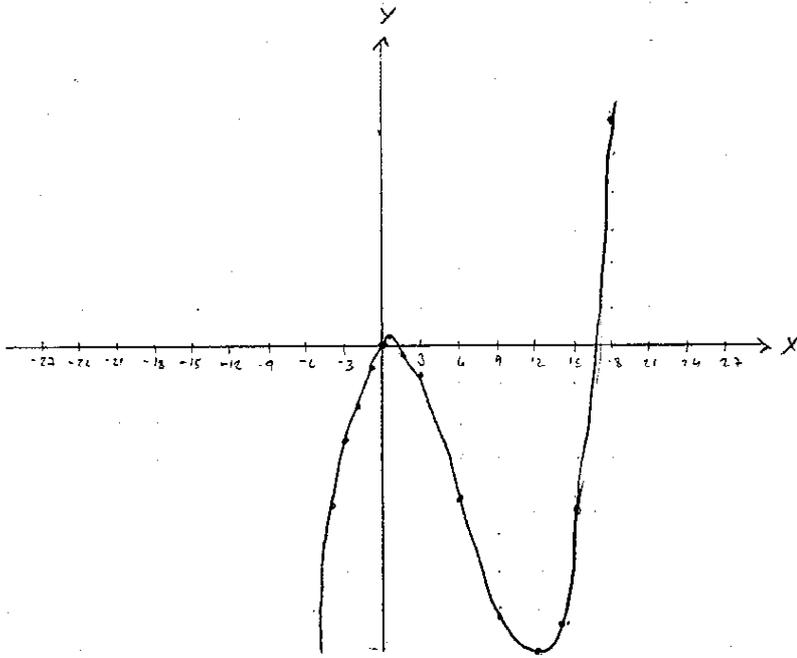
$$Y(3) = 3.92$$

$$Y(3.5) = 0.41$$

$$Y(4) = -8.77$$



$$Y(X) = X - \frac{3}{4}X^2 + \frac{1}{24}X^3$$



תוצאות טבלה 3 מוב $\Sigma 1$ | תוצאה : 2 שלב למטה

$$Y(0) = 0$$

$$Y(1) = 0.291$$

$$Y(-1) = -1.79$$

$$Y(3) = -2.625$$

$$Y(-2) = -5\frac{1}{3}$$

$$Y(6) = -12$$

$$Y(-3) = -10.87$$

$$Y(9) = -21.37$$

$$Y(-4) = -18\frac{2}{3}$$

$$Y(12) = -24$$

$$Y(-5) = -28.95$$

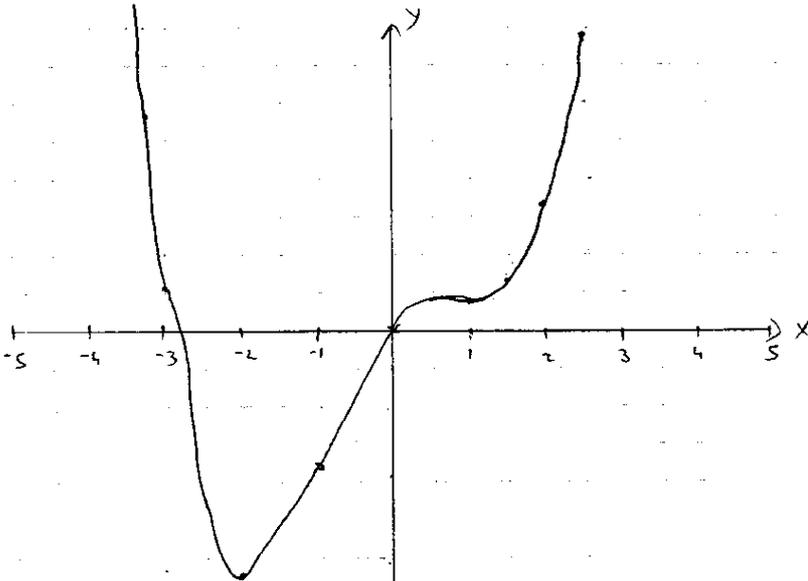
$$Y(14) = -18.667$$

$$Y(18) = 18$$

$$Y(15) = -13.125$$

$$Y(2) = -\frac{2}{3}$$

$$Y(X) = X - \frac{3}{4}X^2 + \frac{1}{24}X^3 + \frac{9}{64}X^4$$



תוצאות טבלה 4 מוב $\Sigma 1$ | תוצאה

$$Y(0) = 0$$

$$Y(-1) = -1.65$$

$$Y(1) = 0.432$$

$$Y(-2) = -3.02$$

$$Y(2) = 1.583$$

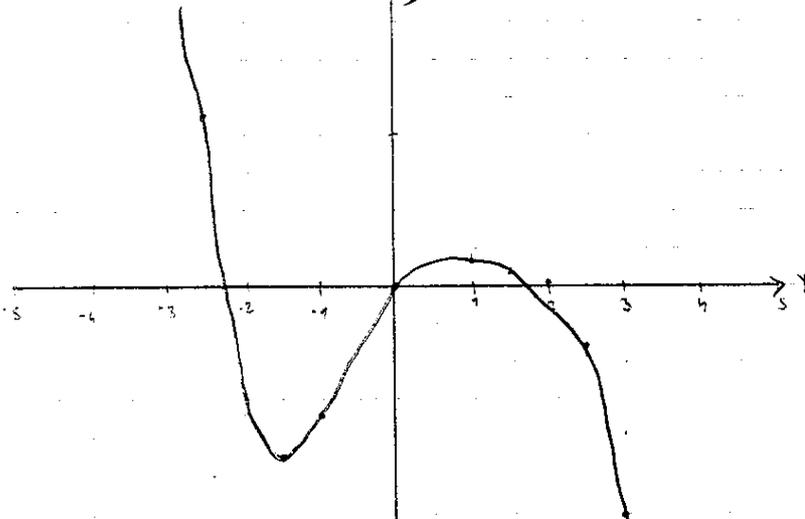
$$Y(-3) = 0.515$$

$$Y(2.5) = 3.95$$

$$Y(-3.5) = 6.62$$

$$Y(1.5) = 0.66$$

$$Y(X) = X - \frac{3}{4}X^2 + \frac{1}{24}X^3 + \frac{9}{64}X^4 - \frac{31}{640}X^5$$



תוצאות טבלה 5 מוב $\Sigma 1$ | תוצאה

$$Y(0) = 0$$

$$Y(-1) = -1.602$$

$$Y(1) = 0.38$$

$$Y(-1.5) = -2.24$$

$$Y(1.5) = 0.29$$

$$Y(-2) = -1.53$$

$$Y(2) = 0.03$$

$$Y(-2.5) = 2.38$$

$$Y(2.5) = 0.77$$

$$Y(3) = -3.004$$

3. $(1-x^2)y'' - xy' + k^2y = 0$ $T_k(x)$ $k=0,1,2,3,4$

$y' = \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}$, $y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$, $y = \sum_{n=0}^{\infty} a_n x^n$ כבר סתם

בגלל זה

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - \sum_{n=2}^{\infty} n(n-1)a_n x^{n-1} - \sum_{n=1}^{\infty} n a_n x^n + k^2 \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2} x^n - \sum_{n=0}^{\infty} n(n-1)a_n x^n - \sum_{n=0}^{\infty} n a_n x^n + \sum_{n=0}^{\infty} k^2 a_n x^n = 0$$

(בגלל זה)

$$(n+2)(n+1)a_{n+2} - n(n-1)a_n - n a_n + k^2 a_n = 0$$

$$(n+2)(n+1)a_{n+2} = [n^2 - n + k^2] a_n$$

$$a_{n+2} = \frac{(n^2 - k^2)a_n}{(n+2)(n+1)}$$

כבר סתם

$n=0$ $a_2 = \frac{-k^2 a_0}{2 \cdot 1}$ $n=1$ $a_3 = \frac{(1-k^2)a_1}{3 \cdot 2}$

$n=2$ $a_4 = \frac{(2^2 - k^2)a_2}{4 \cdot 3} = \frac{(2^2 - k^2)(-k^2)a_0}{4 \cdot 3 \cdot 2 \cdot 1}$ $n=3$ $a_5 = \frac{(3^2 - k^2)a_3}{5 \cdot 4} = \frac{(3^2 - k^2)(1 - k^2)a_1}{5 \cdot 4 \cdot 3 \cdot 2}$

$$a_{2m} = \frac{(2m-2)^2 - k^2 \dots (2^2 - k^2)(-k^2)}{(2m)!} \cdot a_0$$

$$a_{2m+1} = \frac{(2m-1)^2 - k^2 \dots (1 - k^2)}{(2m+1)!} \cdot a_1$$

$$y(x) = \sum_{n=0}^{\infty} a_n x^n = \sum_{m=0}^{\infty} \left[\frac{(2m-2)^2 - k^2 \dots (-k^2)}{(2m)!} a_0 x^{2m} \right] + \sum_{m=0}^{\infty} \left[\frac{(2m-1)^2 - k^2 \dots (1 - k^2)}{(2m+1)!} a_1 x^{2m+1} \right]$$

$$= a_0 \left[1 + \sum_{m=1}^{\infty} \frac{(2m-2)^2 - k^2 \dots (-k^2)}{(2m)!} x^{2m} \right] + a_1 \left[x + \sum_{m=1}^{\infty} \frac{(2m-1)^2 - k^2 \dots (1 - k^2)}{(2m+1)!} x^{2m+1} \right]$$

$$= a_0 y_0^{(k)}(x) + a_1 y_1^{(k)}(x)$$

$T_0(x) = Y_1^{(0)}(x) = 1 + \sum_{m=1}^{\infty} \frac{(2m-2)^2 - 0^2 \dots (-0^2)}{(2m)!} x^{2m} = 1$ כבר סתם

$T_1(x) = Y_2^{(1)}(x) = x + \sum_{m=1}^{\infty} \frac{(2m-1)^2 - 1^2 \dots (1-1^2)}{(2m+1)!} x^{2m+1} = x + \sum_{m=1}^{\infty} \frac{(2m-1)^2 - 1^2 \dots 0}{(2m+1)!} x^{2m+1} = x$

$-T_2(x) = Y_1^{(2)}(x) = 1 + \frac{-2^2}{2!} x^2 + \sum_{m=2}^{\infty} \frac{(2m-2)^2 - 2^2 \dots (2^2 - 2^2)(-2^2)}{(2m)!} x^{2m} = 1 - 2x^2$

$-\frac{1}{3}T_3(x) = Y_2^{(3)}(x) = x + \frac{(1-3^2)}{3!} x^3 + \sum_{m=2}^{\infty} \frac{(2m-1)^2 - 1^2 \dots (3^2 - 3^2)(1-3^2)}{(2m+1)!} x^{2m+1} = x - \frac{4}{3}x^3$

$c \cdot T_4(x) = Y_1^{(4)}(x) = 1 + \frac{-2^2}{2!} x^2 + \frac{(2^2 - 4^2)(-4^2)}{4!} x^4 + \sum_{m=3}^{\infty} \frac{(2m-2)^2 - 4^2 \dots (4^2 - 4^2)(-4^2)}{(2m)!} x^{2m}$