

$$1. (c) \quad \tilde{X} = \begin{pmatrix} 4 & -3 \\ 3 & 4 \end{pmatrix} X$$

$$|A - \lambda I| = \begin{vmatrix} 4-\lambda & -3 \\ 3 & 4-\lambda \end{vmatrix} = (4-\lambda)^2 + 9 = 0$$

$$4-\lambda = \pm 3i$$

$$\lambda_1 = 4+3i \quad \lambda_2 = 4-3i$$

A 50-667 N (3N)

$$(A - \lambda I) X = 0$$

$$0 = \begin{pmatrix} 4-4-3i & -3 \\ 3 & 4-4-3i \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} -3i & -3 \\ 3 & -3i \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} -3X_1i - 3X_2 \\ 3X_1 - 3X_2i \end{pmatrix} = 0$$

$$\begin{cases} -3x_1 + -3x_2 = 0 \\ 3x_1 - 3x_2 = 0 \end{cases} \Rightarrow \begin{cases} -3x_1 + -3x_2 = 0 \\ -3x_1 - 3x_2 = 0 \end{cases} \Rightarrow \begin{aligned} x_1 &= -x_2 \\ -x_1 &= -x_2 \\ x_1 &= x_2 \end{aligned}$$

$$\bar{V}_n = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{ונבנה מ-}\bar{v}$$

• x₂ 26

$$\text{D} = \begin{pmatrix} 4-4+3i & -3 \\ 3 & 4-4+3i \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} 3i & -3 \\ 3 & 3i \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} 3iX_1 - 3X_2 \\ 3X_1 + 3iX_2 \end{pmatrix} = 0$$

$$\begin{cases} 3ix_1 - 3x_2 = 0 \\ 3x_1 + 3ix_2 = 0 \end{cases} \Rightarrow \begin{cases} 3ix_1 - 3x_2 = 0 \\ 3ix_1 - 3x_2 = 0 \end{cases} \Rightarrow x_2 = ix_1$$

$$\bar{V}_2 = \begin{pmatrix} 1 \\ i \end{pmatrix} \quad \text{ויש שורש ריבועי של } -1.$$

$$e^{4t} \bar{U}_1 = e^{(4+3i)t} \bar{V}_1 = e^{4t} (\cos 3t + i \sin 3t) \begin{pmatrix} 1 \\ 1 \end{pmatrix} = e^{4t} \left(\begin{pmatrix} 1 & \cos 3t \\ -\sin 3t & 1 \end{pmatrix} + i \begin{pmatrix} 0 & \sin 3t \\ \cos 3t & 0 \end{pmatrix} \right)$$

הנתקן גניזה נסגרה ב- 20.000 מילון למשך 10 שנים.

$$X(t) = C_1 e^{4t} \begin{pmatrix} -\sin 3t \\ \cos 3t \end{pmatrix} + C_2 e^{-4t} \begin{pmatrix} \cos 3t \\ \sin 3t \end{pmatrix}$$

$$X = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ 1 & -1 & 0 \end{pmatrix} X$$

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & -1 \\ 1 & -\lambda & 0 \\ 1 & -1 & -\lambda \end{vmatrix} \stackrel{\text{row } 1 \rightarrow}{=} (1-\lambda) \begin{vmatrix} -\lambda & 0 \\ -1 & -\lambda \end{vmatrix} - 1 \cdot \begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix} = (1-\lambda) \cdot \lambda^2 - ((-1)+\lambda)$$

$$= \lambda^2 - \lambda^3 + 1 - \lambda = -\lambda^3 + \lambda^2 - \lambda + 1 =$$

$$\Rightarrow (-1)(\lambda^2 + 1)(\lambda - 1) = 0 \Rightarrow \lambda_1 = 1 \\ \lambda_2 = i \\ \lambda_3 = -i$$

$$\frac{-\lambda^2 - 1}{-\lambda^3 + \lambda^2 - \lambda + 1} \lambda - 1 \\ \frac{-\lambda^3 + \lambda^2}{-\lambda + 1} \\ \frac{-\lambda + 1}{0}$$

$$(A - \lambda I)v = 0$$

: מatrix NN ב R^N

$$\begin{pmatrix} 0 & 0 & -1 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} -V_3 \\ V_1 - V_2 \\ V_1 - V_2 - V_3 \end{pmatrix} = 0$$

: $\lambda_1 = 1$

$$\begin{cases} -V_3 = 0 \Rightarrow V_3 = 0 \\ V_1 - V_2 = 0 \Rightarrow V_1 = V_2 \\ V_1 - V_2 - V_3 = 0 \Rightarrow V_1 - V_1 - 0 = 0 \end{cases} \quad \Rightarrow$$

$$\bar{V}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \boxed{\bar{X}_{P_1} = e^t \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}$$

$$\begin{pmatrix} 1-i & 0 & -1 \\ 1 & -i & 0 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} (1-i)V_1 - V_3 \\ V_1 - iV_2 \\ V_1 - V_2 - iV_3 \end{pmatrix} \xrightarrow{R_3 \leftarrow (1-i)R_3 - R_1} \begin{pmatrix} (1-i)V_1 - V_3 \\ -i(1-i)V_2 + V_3 \\ -(i-i)V_2 - i(1-i)V_3 + V_3 \end{pmatrix} \quad : \lambda_2 = i$$

$$= \begin{pmatrix} (1-i)V_1 - V_3 \\ (-1-i)V_2 + V_3 \\ (-1+i)V_2 - iV_3 \end{pmatrix} \xrightarrow{R_3 \leftarrow iR_3} \begin{pmatrix} (1-i)V_1 - V_3 \\ (-1-i)V_2 + V_3 \\ (-1-i)V_2 + iV_3 \end{pmatrix} \xrightarrow{R_3 \leftarrow R_3 - R_2} \begin{pmatrix} (1-i)V_1 - V_3 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \bar{V}_2 = \begin{pmatrix} 1 \\ -1 \\ 1-i \end{pmatrix}$$

$$\bar{X}_{P_2} = e^t \begin{pmatrix} 1 \\ -1 \\ 1-i \end{pmatrix} = (\cos t + i \sin t) \begin{pmatrix} 1 \\ -1 \\ 1-i \end{pmatrix} = \begin{pmatrix} \cos t \\ \sin t \\ \cos t + \sin t \end{pmatrix} + i \begin{pmatrix} \sin t \\ -\cos t \\ -\cos t + \sin t \end{pmatrix} \quad : \text{ריבועי מסינוס וкосינוס}$$

לעומת זה, מטרת התרגיל היא למצוא פתרון כללי עבור שורה שנייה

$$\boxed{\bar{X}(t) = C_1 e^t \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + C_2 \begin{pmatrix} \cos t \\ \sin t \\ \cos t + \sin t \end{pmatrix} + C_3 \begin{pmatrix} \sin t \\ -\cos t \\ -\cos t + \sin t \end{pmatrix}}$$

$$1(e) \quad \bar{X}' = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix} X$$

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 2 & -1 \\ 0 & 1-\lambda & 1 \\ 0 & -1 & 1-\lambda \end{vmatrix} = (1-\lambda) \begin{vmatrix} 1-\lambda & 1 \\ -1 & 1-\lambda \end{vmatrix} = (1-\lambda)[(1-\lambda)^2 + 1] \quad : A \text{ is } 3 \times 3 \text{ in } \mathbb{R}$$

$$= (1-\lambda)[\lambda^2 - 2\lambda + 1 + 1] = (1-\lambda)[\lambda^2 - 2\lambda + 2] \Rightarrow \lambda_1 = 1 \quad \lambda_{2,3} = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} \quad \lambda_2 = 1+i \quad \lambda_3 = 1-i$$

: now find \vec{v}_1 in \mathbb{C}^3

$$(A - \lambda_1 I) \vec{v} = 0$$

: $\lambda_1 = 1$

$$\begin{pmatrix} 0 & 2 & -1 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 2V_2 - V_3 \\ V_2 \\ -V_2 \end{pmatrix} = 0$$

$$\begin{cases} 2V_2 - V_3 = 0 \\ V_2 = 0 \\ -V_2 = 0 \end{cases} \Rightarrow V_3 = 0 \Rightarrow \vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \boxed{\bar{X}_{P_1} = e^t \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}$$

$$\begin{pmatrix} i & 2 & -1 \\ 0 & i & 1 \\ 0 & -1 & i \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} iV_1 + 2V_2 - V_3 \\ iV_2 + V_3 \\ -V_2 + iV_3 \end{pmatrix} \xrightarrow{R_3 \leftrightarrow iR_3} \begin{pmatrix} iV_1 + 2V_2 - V_3 \\ iV_2 + V_3 \\ iV_2 + V_3 \end{pmatrix} \rightarrow \begin{pmatrix} iV_1 + 2V_2 - V_3 \\ iV_2 + V_3 \\ 0 \end{pmatrix} \quad : \lambda_3 = 1$$

$$\begin{cases} iV_1 + 2V_2 - V_3 = 0 \\ iV_2 + V_3 = 0 \end{cases} \Rightarrow V_3 = -iV_2 \Rightarrow iV_1 + (2+i)V_2 = 0 \Rightarrow (2+i)V_2 = -iV_1 \quad V_1 = -\frac{2+i}{i}V_2$$

$$\Rightarrow V_2 = 1 \quad V_3 = -i \quad V_1 = (2+i)$$

$$\bar{X}_{P_3} = e^{t(1+i)} \begin{pmatrix} 2i-1 \\ 1 \\ -i \end{pmatrix} = e^t (cost + isint) \begin{pmatrix} 2i-1 \\ 1 \\ -i \end{pmatrix} = e^t \begin{pmatrix} -cost - 2isint \\ cost \\ sint \end{pmatrix} + e^t i \begin{pmatrix} 2cost - sint \\ sint \\ -cost \end{pmatrix}$$

: now find \vec{v}_2 in \mathbb{C}^3 so that \bar{X}_{P_2} is a linear combination of \bar{X}_{P_1} and \bar{X}_{P_3}

$$\boxed{X(t) = C_1 e^t \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + C_2 e^t \begin{pmatrix} -cost - 2isint \\ cost \\ sint \end{pmatrix} + C_3 e^t \begin{pmatrix} 2cost - sint \\ sint \\ -cost \end{pmatrix}}$$

$$2.(c) \quad \bar{X}' = \begin{pmatrix} -5 & 1 \\ 4 & -2 \end{pmatrix} \bar{X} \quad X(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{aligned} |A - \lambda I| &= \begin{vmatrix} -5-\lambda & 1 \\ 4 & -2-\lambda \end{vmatrix} = (-5-\lambda)(-2-\lambda) - 4 \\ &= 10 + 2\lambda + 5\lambda + \lambda^2 - 4 = \lambda^2 + 7\lambda + 6 \Rightarrow \lambda_{1,2} = \frac{-7 \pm \sqrt{49-24}}{2} = \frac{-7 \pm 5}{2} \begin{cases} \lambda_1 = -6 \\ \lambda_2 = -1 \end{cases} \end{aligned}$$

$$0 = \begin{pmatrix} -5+6 & 1 \\ 4 & -2+6 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 4 & 4 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} V_1 + V_2 \\ 4V_1 + V_2 \cdot 4 \end{pmatrix} \rightarrow \begin{pmatrix} V_1 + V_2 \\ 0 \end{pmatrix} \begin{cases} \lambda_1 \text{ is } 6 \\ V_1 = -V_2 \end{cases}$$

$$\bar{V}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \Rightarrow \boxed{\bar{X}_{P_1} = e^{-6t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}}$$

$$0 = \begin{pmatrix} -5+1 & 1 \\ 4 & -2+1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} -4 & 1 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} -4V_1 + V_2 \\ -4V_1 + V_2 \end{pmatrix} \rightarrow \begin{pmatrix} -4V_1 + V_2 \\ 0 \end{pmatrix} \begin{cases} \lambda_2 \text{ is } 2 \\ V_1 = \frac{1}{4}V_2 \end{cases}$$

$$\bar{V}_2 = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \Rightarrow \boxed{\bar{X}_{P_2} = e^{-t} \begin{pmatrix} 1 \\ 4 \end{pmatrix}}$$

$$\boxed{\bar{X}(t) = C_1 e^{-6t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + C_2 e^{-t} \begin{pmatrix} 1 \\ 4 \end{pmatrix}}$$

$$\bar{X}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix} = C_1 e^0 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + C_2 e^0 \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} C_1 + C_2 \\ -C_1 + 4C_2 \end{pmatrix}$$

$$\begin{cases} C_1 + C_2 = 1 \\ -C_1 + 4C_2 = 2 \end{cases} \Rightarrow 4C_2 - 2 + C_2 = 1 \Rightarrow 5C_2 = 3 \Rightarrow C_2 = \frac{3}{5} \Rightarrow C_1 = \frac{2}{5}$$

$$\boxed{\bar{X}(t) = e^{-6t} \begin{pmatrix} \frac{2}{5} \\ -\frac{2}{5} \end{pmatrix} + e^{-t} \begin{pmatrix} \frac{3}{5} \\ \frac{12}{5} \end{pmatrix}}$$

$$2. (2) \quad \bar{X}' = \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix} \bar{X} \quad \bar{X}(0) = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$\begin{vmatrix} 3-\lambda & -9 \\ 4 & -3-\lambda \end{vmatrix} = (3-\lambda)(-3-\lambda) + 36 = -9 + 2\lambda - 3\lambda + \lambda^2 + 36 = \lambda^2 + 2\lambda = 0$$

$$\lambda^2 = -2\lambda \Rightarrow \lambda_{1,2} = \pm \sqrt{2}\lambda i = \pm 3\sqrt{3}i$$

$$\lambda_1 = 3\sqrt{3}i, \lambda_2 = -3\sqrt{3}i \quad (\text{PN})$$

$$(A - \lambda I)v = 0$$

$$0 = \begin{pmatrix} 3-3\sqrt{3}i & -9 \\ 4 & -3-3\sqrt{3}i \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \Rightarrow \begin{pmatrix} 3-3\sqrt{3}i & -9 & | & 0 \\ 4 & -3\sqrt{3}i & | & 0 \end{pmatrix} \xrightarrow[R_2 \leftarrow \frac{3-3\sqrt{3}i}{4} R_2]{R_1} \begin{pmatrix} 3-3\sqrt{3}i & -9 & | & 0 \\ 3-\sqrt{3}i & -\frac{36}{4} & | & 0 \end{pmatrix} \xrightarrow[R_2 \leftarrow R_2 - R_1]{R_1 \leftarrow \frac{R_1}{3}} \begin{pmatrix} 1-\sqrt{3}i & -3 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix}$$

$$\bar{v}_1 = \begin{pmatrix} \frac{3}{1-\sqrt{3}i} s \\ s \end{pmatrix} = \begin{pmatrix} (3+3\sqrt{3}i)s \\ s \end{pmatrix} \Rightarrow \bar{v}_1 = \begin{pmatrix} 3(1+\sqrt{3}i) \\ 1 \end{pmatrix}$$

$$0 = \begin{pmatrix} 3(1+\sqrt{3}i) & -9 \\ 4 & -3+3\sqrt{3}i \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \Rightarrow \begin{pmatrix} 3(1+\sqrt{3}i) & -9 & | & 0 \\ 4 & -3(1-\sqrt{3}i) & | & 0 \end{pmatrix} \xrightarrow[R_2 \leftarrow \frac{1+\sqrt{3}i}{4} R_2]{R_1 \leftarrow \frac{R_1}{3}} \begin{pmatrix} 1+\sqrt{3}i & -3 & | & 0 \\ 1+\sqrt{3}i & -3 & | & 0 \end{pmatrix} \xrightarrow[R_2 \leftarrow R_2 - R_1]{R_1} \begin{pmatrix} 1+\sqrt{3}i & -3 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix}$$

$$\bar{v}_2 = \begin{pmatrix} \frac{1}{1+\sqrt{3}i} s \\ s \end{pmatrix} = \begin{pmatrix} (3-3\sqrt{3}i)s \\ s \end{pmatrix} \Rightarrow \bar{v}_2 = \begin{pmatrix} 3(1-\sqrt{3}i) \\ 1 \end{pmatrix}$$

$$\bar{x}_{p_1} = e^{(3\sqrt{3}it)} \bar{v}_1 = (\cos 3\sqrt{3}t + i \sin 3\sqrt{3}t) \begin{pmatrix} 3(1+\sqrt{3}i) \\ 1 \end{pmatrix}$$

$$\boxed{\bar{x}_{p_1} = \begin{pmatrix} 3\cos(3\sqrt{3}t) - 3\sqrt{3}\sin(3\sqrt{3}t) \\ \cos(3\sqrt{3}t) \end{pmatrix} + i \begin{pmatrix} 3\sqrt{3}\cos(3\sqrt{3}t) + 3\sin(3\sqrt{3}t) \\ \sin(3\sqrt{3}t) \end{pmatrix}}$$

הנימוקים הנדרשים בפתרון הינו:

$$\bar{X}(t) = C_1 \begin{pmatrix} 3\cos(3\sqrt{3}t) - 3\sqrt{3}\sin(3\sqrt{3}t) \\ \cos(3\sqrt{3}t) \end{pmatrix} + C_2 \begin{pmatrix} 3\sqrt{3}\cos(3\sqrt{3}t) + 3\sin(3\sqrt{3}t) \\ \sin(3\sqrt{3}t) \end{pmatrix}$$

$$\bar{X}(0) = \begin{pmatrix} C_1 \cdot 3\cos(0) - 3\sqrt{3}\sin(0) + C_2 \cdot 3\sqrt{3}\cos(0) + 3\sin(0) \\ C_1 \cos(0) + C_2 \sin(0) \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$\begin{cases} 3C_1 + 3\sqrt{3}C_2 = 2 \\ C_1 = -4 \end{cases} \Rightarrow -12 + 3\sqrt{3}C_2 = 2 \Rightarrow \boxed{C_2 = \frac{14}{3\sqrt{3}}}$$

הנימוקים הנדרשים בפתרון הינו:

$$\boxed{\bar{X}(t) = -4 \begin{pmatrix} 3\cos(3\sqrt{3}t) - 3\sqrt{3}\sin(3\sqrt{3}t) \\ \cos(3\sqrt{3}t) \end{pmatrix} + \frac{14}{3\sqrt{3}} \begin{pmatrix} 3\sqrt{3}\cos(3\sqrt{3}t) + 3\sin(3\sqrt{3}t) \\ \sin(3\sqrt{3}t) \end{pmatrix}}$$

$$2.(b) \quad \bar{X} = \begin{pmatrix} 7 & 1 \\ -4 & 3 \end{pmatrix} \bar{x} \quad X(0) = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

$$\begin{vmatrix} 7-\lambda & 1 \\ -4 & 3-\lambda \end{vmatrix} = (7-\lambda)(3-\lambda) + 4 = 21 - 3\lambda - 7\lambda + \lambda^2 + 4 \quad \text{רנ'ון } 66 \text{ (ב)}$$

$$= \lambda^2 - 10\lambda + 25 = 0 \Rightarrow \lambda_{1,2} = \frac{10 \pm \sqrt{100 - 4 \cdot 25}}{2} \Rightarrow \lambda_{1,2} = 5$$

$$\begin{pmatrix} 7-5 & 1 \\ -4 & 3-5 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -4 & -2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 2v_1 + v_2 \\ 0 \end{pmatrix} \Rightarrow \bar{v}_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \quad : \text{ט'ג נס (ב)}$$

$$\begin{pmatrix} 7-5 & 1 \\ -4 & 3-5 \end{pmatrix} \begin{pmatrix} \bar{v}_1 \\ \bar{v}_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \Rightarrow \left| \begin{array}{cc|c} 2 & 1 & 1 \\ -4 & -2 & -2 \end{array} \right| \xrightarrow[R_1 \leftrightarrow R_2]{R_2 \leftarrow R_2 - 2R_1} \left| \begin{array}{cc|c} 2 & 1 & 1 \\ 0 & 0 & 0 \end{array} \right| \quad 0 \geq \bar{v}_2 \quad \text{(ב)}$$

$$\Rightarrow 2\bar{v}_1 + \bar{v}_2 = 1 \Rightarrow \bar{v}_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\bar{x}_{p_1}(t) = \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{st}$$

ענ'ו 106 → 108 → 108

$$\bar{x}_{p_2}(t) = (t \cdot \bar{v}_1 + \bar{v}_2) e^{st} = (t \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}) e^{st} = \begin{pmatrix} t+2 \\ -2t-1 \end{pmatrix} e^{st} \quad \text{ענ'ו 108}$$

$$\boxed{\bar{x}(t) = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{st} + C_2 \begin{pmatrix} t+2 \\ -2t-1 \end{pmatrix} e^{st}}$$

ולא נזכיר

$$3. (a) \quad X(t) = Y \quad X_1'(t) = X_2 \\ Y'(t) = \frac{2}{t^2} X \quad \Rightarrow \quad X_2'(t) = \frac{2}{t^2} X_1$$

$$\bar{X}' = \begin{pmatrix} 0 & 1 \\ \frac{2}{t^2} & 0 \end{pmatrix} \bar{X}$$

$$\begin{vmatrix} -\lambda & 1 \\ \frac{2}{t^2} & -\lambda \end{vmatrix} = \lambda^2 - \frac{2}{t^2} = 0 \Rightarrow \lambda^2 = \frac{2}{t^2} \Rightarrow \lambda = \pm \frac{\sqrt{2}}{t}$$

$$0 = \begin{pmatrix} -\frac{\sqrt{2}}{t} & 1 \\ \frac{2}{t^2} & -\frac{\sqrt{2}}{t} \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} \Rightarrow \begin{pmatrix} -\frac{\sqrt{2}}{t} & 1 & 0 \\ \frac{2}{t^2} & -\frac{\sqrt{2}}{t} & 0 \end{pmatrix} \xrightarrow{R_2 \leftarrow \frac{t}{\sqrt{2}} R_2} \begin{pmatrix} -\frac{\sqrt{2}}{t} & 1 & 0 \\ \frac{\sqrt{2}}{t} & 1 & 0 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} -\frac{\sqrt{2}}{t} & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\bar{U}_1(t) = \begin{pmatrix} \frac{t}{\sqrt{2}} S \\ S \end{pmatrix} \Rightarrow \boxed{\bar{U}_1(t) = \begin{pmatrix} \frac{t}{\sqrt{2}} \\ 1 \end{pmatrix}}$$

$$0 = \begin{pmatrix} \frac{\sqrt{2}}{t} & 1 \\ \frac{2}{t^2} & \frac{\sqrt{2}}{t} \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{\sqrt{2}}{t} & 1 & 0 \\ \frac{2}{t^2} & \frac{\sqrt{2}}{t} & 0 \end{pmatrix} \xrightarrow{R_2 \leftarrow \frac{t}{\sqrt{2}} R_2} \begin{pmatrix} \frac{\sqrt{2}}{t} & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} \frac{\sqrt{2}}{t} & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \lambda_2 = 0$$

$$\boxed{\bar{U}_2(t) = \begin{pmatrix} -\frac{t}{\sqrt{2}} \\ 1 \end{pmatrix}}$$

$$X_{P_1}(t) = e^{\frac{\sqrt{2}}{t} \cdot t} \bar{U}_1(t) = e^{\sqrt{2}} \begin{pmatrix} \frac{t}{\sqrt{2}} \\ 1 \end{pmatrix}$$

$$X_{P_2}(t) = e^{-\frac{\sqrt{2}}{t} \cdot t} \bar{U}_2(t) = e^{-\sqrt{2}} \begin{pmatrix} -\frac{t}{\sqrt{2}} \\ 1 \end{pmatrix}$$

$$\boxed{\bar{X}(t) = C_1 e^{\sqrt{2}} \begin{pmatrix} \frac{t}{\sqrt{2}} \\ 1 \end{pmatrix} + C_2 e^{-\sqrt{2}} \begin{pmatrix} -\frac{t}{\sqrt{2}} \\ 1 \end{pmatrix}}$$

$$3.(o) \quad \begin{cases} \frac{dx}{dt} = ax + y \\ \frac{dy}{dt} = -x + ay \end{cases} \Rightarrow \begin{cases} x_1' = ax_1 + x_2 \\ x_2' = -x_1 + ax_2 \end{cases}$$

$$\bar{X}'(t) = \begin{pmatrix} a & 1 \\ -1 & a \end{pmatrix} \bar{X}(t)$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} a-\lambda & 1 \\ -1 & a-\lambda \end{vmatrix} = (a-\lambda)^2 + 1 = 0$$

$$(a-\lambda)^2 = -1$$

$$(a-\lambda) = \pm i$$

$$\boxed{\lambda_{1,2} = a \pm i}$$

$$(A - \lambda I)V = 0$$

$$0 = \begin{pmatrix} a-a-i & 1 \\ -1 & a-a-i \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \Rightarrow \begin{pmatrix} -i & 1 \\ -1 & -i \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 + i} \begin{pmatrix} -i & 1 \\ -i & 0 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} -i & 1 \\ 0 & 0 \end{pmatrix} \quad \text{for } \lambda_1$$

$$\boxed{\bar{V}_1(t) = \begin{pmatrix} -i \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ i \end{pmatrix}}$$

$$0 = \begin{pmatrix} a-a+i & 1 \\ -1 & a-a+i \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \Rightarrow \begin{pmatrix} i & 1 \\ -1 & i \end{pmatrix} \xrightarrow{R_2 \leftarrow -iR_2} \begin{pmatrix} i & 1 \\ i & 0 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} i & 1 \\ 0 & 0 \end{pmatrix} \quad \text{for } \lambda_2$$

$$\boxed{\bar{V}_2(t) = \begin{pmatrix} i \\ 1 \end{pmatrix}}$$

$$\bar{X}_{P_1}(t) = e^{at+i} \bar{V}_1 = e^a (\cos t + i \sin t) \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$\bar{X}_{P_1}(t) = e^a \left[\begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} + i \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} \right]$$

לפיה פונקציית ריבועית מוגדרת כפונקציית מרום של פונקציית מרום

$$\boxed{\bar{X}(t) = c_1 e^a \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} + c_2 e^a \begin{pmatrix} \sin t \\ \cos t \end{pmatrix}}$$