

$$x^2 y'' - x(x+2)y' + (x+2)y = 0 \quad x_0 = 0$$

✓  $x^2 y'' - x(x+2)y' + (x+2)y = 0$  ליד  $x_0 = 0$  נכנס

$$x \frac{Q(x)}{P(x)} = -k \cdot \frac{x(x+2)}{x^2} = -(x+2) \xrightarrow{x \rightarrow 0} -2 \quad x^2 \frac{R(x)}{P(x)} = -x^2 \frac{(x+2)}{x^2} = -(x+2) \xrightarrow{x \rightarrow 0} -2$$

$$y(x) = \sum_{n=0}^{\infty} a_n x^{n+r} \quad \text{נכנס פתרון מניח}$$

$$y'(x) = \sum_{n=0}^{\infty} a_n (n+r) x^{n+r-1} \quad y''(x) = \sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r-2}$$

$$x^2 y'' = \sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r}$$

$$x y' = \sum_{n=0}^{\infty} a_n (n+r) x^{n+r}$$

$$\sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r} - \sum_{n=0}^{\infty} a_n (n+r) x^{n+r+1} - \sum_{n=0}^{\infty} 2a_n (n+r) x^{n+r} + \sum_{n=0}^{\infty} a_n x^{n+r+1} + \sum_{n=0}^{\infty} 2a_n x^{n+r} = 0$$

$$\sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r} - \sum_{n=0}^{\infty} a_n (n+r-1) x^{n+r+1} - \sum_{n=0}^{\infty} 2(n+r-1) x^{n+r} a_n = 0$$

$$\sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r} - \sum_{n=1}^{\infty} a_{n-1} (n+r-2) x^{n+r} - \sum_{n=0}^{\infty} 2(n+r-1) x^{n+r} a_n = 0$$

$$a_0 r(r-1) - 2a_0(r-1) = 0$$

$$a_0 [(r-1)(r-2)] = 0$$

$$r_1 = 1, r_2 = 2 \quad \text{כך נקבל } a_0 \neq 0 \quad \text{כך נניח}$$

$$a_n [(n+r)(n+r-1) - 2(n+r-1)] = a_{n-1} (n+r-2)$$

$$a_n [(n+r-1)(n+r-2)] = a_{n-1} (n+r-2)$$

$$a_n = \frac{a_{n-1}}{(n+r-1)}$$

$$a_1 = \frac{a_0}{1+1-1} = a_0$$

$$a_2 = \frac{a_1}{2+1-1} = \frac{1}{2} a_1 = \frac{1}{2!} a_0$$

$$a_3 = \frac{a_2}{3+1-1} = \frac{1}{3} a_2 = \frac{1}{3!} a_0$$

$$a_n = \frac{1}{n!} a_0$$

$$y_1(x) = x^1 \left[ a_0 \sum_{n=0}^{\infty} \frac{1}{n!} x^n \right]$$

כך נקבל

$$a_1 = \frac{a_0}{1+2-1} = \frac{1}{2} a_0$$

$$a_2 = \frac{a_1}{2+2-1} = \frac{1}{3} a_1 = \frac{1}{3!} a_0$$

$$a_3 = \frac{a_2}{3+2-1} = \frac{1}{4} a_2 = \frac{1}{4!} a_0$$

$$a_n = \frac{1}{(n+1)!} a_0$$

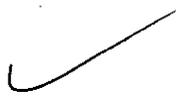
$$y_2(x) = x^2 \left[ a_0 \cdot \sum_{n=0}^{\infty} \frac{1}{(n+1)!} x^n \right]$$

סדר קטן

$$y(x) = C_1 x \sum_{n=0}^{\infty} \frac{1}{n!} x^n + C_2 x^2 \sum_{n=0}^{\infty} \frac{1}{(n+1)!} x^n$$

סדר הסימון הלא

$$y(x) = C_1 x e^x + C_2 x^2 \sum_{n=0}^{\infty} \frac{1}{(n+1)!} x^n$$



1. b.  $2x^2 y'' + (3x - 2x^2) y' - (x+1)y = 0$

$x^2 y'' + (\frac{3}{2}x - x^2) y' - \frac{1}{2}(x+1)y = 0$

$x \cdot \frac{(\frac{3}{2}x - x^2)}{x^2} = x^2 \cdot \frac{(\frac{3}{2} - x)}{x^2} = \frac{3}{2} - x$        $-x^2 \cdot \frac{\frac{1}{2}(x+1)}{x^2} = -\frac{1}{2}(x+1)$

$y = \sum_{n=0}^{\infty} a_n x^{n+r} \Rightarrow y' = \sum_{n=0}^{\infty} a_n (n+r) x^{n+r-1} \Rightarrow y'' = \sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r-2}$

$\sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r} + \sum_{n=0}^{\infty} \frac{3}{2} a_n (n+r) x^{n+r} - \sum_{n=0}^{\infty} a_n (n+r) x^{n+r+1} - \sum_{n=0}^{\infty} \frac{1}{2} a_n x^{n+r+1} = 0$

$\sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r} + \sum_{n=0}^{\infty} a_n \frac{1}{2} (3n+3r-1) x^{n+r} - \sum_{n=0}^{\infty} a_n (n+r+\frac{1}{2}) x^{n+r+1} = 0$

$\sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r} + \sum_{n=0}^{\infty} a_n \frac{1}{2} (3n+3r-1) x^{n+r} - \sum_{n=1}^{\infty} a_{n-1} (n+r-\frac{1}{2}) x^{n+r} = 0$

$a_0 [r(r-1) + \frac{1}{2}(3r-1)] = 0$       ברוך  $n=0$  ר' 26

$r^2 - r + \frac{3}{2}r - \frac{1}{2} = 0 \Rightarrow r^2 + \frac{1}{2}r - \frac{1}{2} = 0$        $a_0 \neq 0$  נניח ברוך

$r_{1,2} = \frac{-\frac{1}{2} \pm \sqrt{\frac{1}{4} + 2}}{2} = \frac{-\frac{1}{2} \pm \sqrt{\frac{9}{4}}}{2} = \frac{-\frac{1}{2} \pm \frac{3}{2}}{2} \begin{cases} r_1 = \frac{1}{2} \\ r_2 = -1 \end{cases}$

$a_n [(n+r)(n+r-1) + \frac{1}{2}(3n+3r-1)] - a_{n-1} (n+r-\frac{1}{2}) = 0$       ברוך  $n$  ר' 26

$a_n = \frac{(n+r-\frac{1}{2})}{[(n+r)(n+r-1) + \frac{1}{2}(3n+3r-1)]} a_{n-1}$

$a_n = \frac{(n+\frac{1}{2}-\frac{1}{2})}{[(n+\frac{1}{2})(n+\frac{1}{2}-1) + \frac{1}{2}(3n+\frac{3}{2}-1)]} a_{n-1}$       ברוך  $r_1 = \frac{1}{2}$  ר' 31

$= \frac{n}{[(n+\frac{1}{2})(n-\frac{1}{2}) + \frac{1}{2}(3n+\frac{1}{2})]} a_{n-1} = \frac{n \cdot a_{n-1}}{n^2 - \frac{1}{4} + \frac{3}{2}n + \frac{1}{4}} = \frac{n a_{n-1}}{n(n+\frac{3}{2})} = \frac{a_{n-1}}{(n+\frac{3}{2})}$

$a_1 = \frac{a_0}{(1+\frac{3}{2})} = \frac{a_0}{\frac{5}{2}} = \frac{2}{5} \cdot a_0$

$a_2 = \frac{a_1}{(2+\frac{3}{2})} = \frac{a_1}{\frac{7}{2}} = \frac{2}{7} \cdot a_1 = \frac{2}{7} \cdot \frac{2}{5} \cdot a_0$

$a_3 = \frac{a_2}{(3+\frac{3}{2})} = \frac{2}{9} a_2 = \frac{2}{9} \cdot \frac{2}{7} \cdot \frac{2}{5} \cdot a_0$

$a_n = \frac{(2)^n \cdot a_0}{(2(n+5)) \dots 5}$

$y(x) = x^{\frac{1}{2}} \left[ 1 + \sum_{n=1}^{\infty} \frac{(2)^n}{(2(n+5)) \dots 5} x^n \right]$

ברוך  $a_0 = 1$  נניח

סדרה  $r = -1$  : (1,6) קבוע

$$a_n = \frac{(n-1-\frac{1}{2})}{[(n-1)(n-2) + \frac{1}{2}(3n-3-1)]} a_{n-1}$$

$$= \frac{(n-\frac{3}{2})}{n^2 - 2n - n + \frac{3}{2}n} a_{n-1} = \frac{(n-\frac{3}{2})}{(n^2 - \frac{3}{2}n)} a_{n-1} = \frac{(n-\frac{3}{2})}{n(n-\frac{3}{2})} a_{n-1} = \frac{a_{n-1}}{n}$$

$$a_1 = \frac{a_0}{1} = a_0$$

$$a_2 = \frac{1}{2} \cdot a_1 = \frac{1}{2 \cdot 1} \cdot a_0$$

$$a_3 = \frac{1}{3} a_2 = \frac{1}{3 \cdot 2 \cdot 1} a_0$$

$$a_n = \frac{1}{n!} a_0$$

לכן

סדרה  $a_0 = 1$  נכונה

$$Y_2(x) = x^{-1} \left[ 1 + \sum_{n=1}^{\infty} \frac{1}{n!} x^n \right] = \frac{e^x}{x}$$

סדרה  $a_0 = 1$  נכונה

$$Y = C_1 \cdot Y_1(x) + C_2 Y_2(x)$$

$$Y = C_1 \left[ x^{\frac{1}{2}} \left( 1 + \sum_{n=1}^{\infty} \frac{(2)^n}{(n+1) \cdot 5} x^n \right) \right] + C_2 \frac{e^x}{x}$$



וזהו

1.c.  $x(x-1)y'' - xy' + y = 0$

נניח  $x_0 = 0$  ב' נחשב את  $(x_0 - 1)y''$  ו'  $(x_0 - 1)y'$

$x \cdot \frac{Q(x)}{P(x)} = x \cdot \frac{(-x)}{(x-1)x} = -\frac{x}{x-1} \Rightarrow$  נחשב את  $x_0 = 0$  ו' נחשב את  $(x_0 - 1)y''$

$x^2 \cdot \frac{R(x)}{P(x)} = x^2 \cdot \frac{1}{x(x-1)} = \frac{x}{x-1} \Rightarrow$  נחשב את  $x_0 = 0$  ו' נחשב את  $(x_0 - 1)y'$

$y(x) = \sum_{n=0}^{\infty} a_n x^{n+r} \Rightarrow y' = \sum_{n=0}^{\infty} a_n (n+r) x^{n+r-1} \Rightarrow y'' = \sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r-2}$

נכנס את זה לתוך המשוואה

$x(x-1) \sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r-2} - \sum_{n=0}^{\infty} a_n (n+r) x^{n+r-1} + \sum_{n=0}^{\infty} a_n x^{n+r} = 0$

$\sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r} - \sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r-1} - \sum_{n=0}^{\infty} a_n (n+r) x^{n+r} + \sum_{n=0}^{\infty} a_n x^{n+r} = 0$

$a_0 \cdot r(r-1) \cdot x^{r-1} + \sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r} - \sum_{n=0}^{\infty} a_{n+1} (n+r+1)(n+r) x^{n+r} - \sum_{n=0}^{\infty} a_n (n+r) x^{n+r} + \sum_{n=0}^{\infty} a_n x^{n+r} = 0$

$a_0 \cdot r(r-1) = 0 \Rightarrow r_1 = 0, r_2 = 1$   $X^{n+r}$  נ'כנס

$r_2 = 1$  נ'בחר את  $r = 1$  ו' נחשב את  $a_n$

$x^r :$   $a_0 [r(r-1) - r+1] - a_1 [(r+1)r] = 0$

$a_0 [1(1-1) - 1+1] - a_1 [(1+1)1] = 0 \Rightarrow 2a_1 = 0 \cdot a_0 \Rightarrow a_1 = 0$

$a_n [(n+r)(n+r-1) - (n+r) + 1] - a_{n+1} (n+r+1)(n+r) = 0$  נ'כנס  $n$  ו' נ'כנס

$a_{n+1} = \frac{[(n+r)(n+r-2) + 1]}{(n+r+1)(n+r)} a_n = \frac{[(n+1)(n-1) + 1]}{(n+2)(n+1)} a_n = \frac{n^2}{(n+2)(n+1)} a_n$

$a_n = 0$  ו'  $a_{n+1} = 0, n > 1$  נ'כנס

$y_2(x) = x \left[ \sum_{n=0}^{\infty} a_n x^n \right] = x \cdot a_0$

$y_2(x) = a_0 x$

$y_2(x) = x$



נ'בחר את  $r = 1$  ו' נחשב את  $a_n$

$a_0 = 1$  ו' נ'כנס

$a_{n+1}(r) = \frac{(n+r-1)^2}{(n+r+1)(n+r)} a_n$

$a_1(r) = \frac{(r-1)^2}{r(r+1)} a_0$

$a_2(r) = \frac{r^2}{(r+2)(r+1)} a_1 = \frac{r^2 \cdot (r-1)^2}{(r+2)(r+1)r(r+1)} a_0$

$a_3(r) = \frac{(r+1)^2}{(r+3)(r+2)} a_2 = \frac{(r+1)^2 \cdot r(r-1)^2}{(r+3)(r+2)(r+1)r(r+1)} a_0$

$a_4(r) = \frac{(r+2)^2}{(r+4)(r+3)} a_3 = \frac{(r+2)^2 \cdot r(r-1)^2}{(r+4)(r+3)(r+2)(r+1)r(r+1)} a_0$



$$a_n(r) = \frac{r \cdot (r-1)^2}{(r+n)(r+n-1)^2} a_0$$

1/6W

$$C_n(r=0) = \frac{d}{dr} [r \cdot a_n(r)] = r \cdot a_n'(r) + 1 \cdot a_n(r) \Big|_{r=0} = 0 + a_n(0)$$

$$C_n = \frac{0 \cdot (-1)^2}{n(n-1)^2} a_0 \Rightarrow \boxed{C_n = 0}$$

a r=0 (3W)

$$a = \lim_{r \rightarrow 0} (r-0) a_1(r) = \lim_{r \rightarrow 0} r \cdot \frac{r \cdot (r-1)^2}{(r+1) \cdot r^2} = \lim_{r \rightarrow 0} \frac{(r-1)^2}{r+1} \rightarrow \frac{(-1)^2}{1} = \underline{1}$$

b 6p) 5no

$$Y_1(x) = x^0 [1 + \sum_{n=1}^{\infty} C_n(0) x^n] + 1 \cdot Y_2(x) \cdot \ln|x|$$

$$Y_1(x) = 1 + a_0 x \cdot \ln|x|$$

$$\boxed{Y_1(x) = 1 + x \ln|x|}$$



10/6 a0=1 1no

$$\boxed{Y(x) = C_1 (1 + x \ln|x|) + C_2 x}$$



1/6n 1/2no 5no

$$1.d.(*) \quad x^2 y'' + xy' + (4x^2 - \frac{1}{4})y = 0$$

אנחנו מחפשים פתרונות מהצורה  $y = x^r$  (כאן  $x_0 = 0$ )

$$x \cdot \frac{Q(x)}{P(x)} = x \cdot \frac{x}{x^2} = 1$$

$$x^2 \cdot \frac{R(x)}{P(x)} = \frac{4x^2 - \frac{1}{4}}{x^2} = 4x^2 - \frac{1}{4}$$

$$y = \sum_{n=0}^{\infty} a_n x^{n+r} \Rightarrow y' = \sum_{n=0}^{\infty} a_n (n+r) x^{n+r-1} \Rightarrow y'' = \sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r-2}$$

$$\sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r} = \sum_{n=0}^{\infty} a_n (n+r) x^{n+r} + \sum_{n=0}^{\infty} 4a_n x^{n+r+2} - \sum_{n=0}^{\infty} \frac{1}{4} a_n x^{n+r} = 0$$

$$\sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r} + \sum_{n=0}^{\infty} a_n (n+r) x^{n+r} + \sum_{n=2}^{\infty} 4a_{n-2} x^{n+r} - \sum_{n=0}^{\infty} \frac{1}{4} a_n x^{n+r} = 0$$

נשווה מקדמים  $x^{n+r}$  ונצטרף את המונחים

$$\begin{array}{l} x^r \\ x^{r+1} \end{array} \left\{ \begin{array}{l} a_0 [(r-1)r + r - \frac{1}{4}] = a_0 [r^2 - \frac{1}{4}] = 0 \\ a_1 [(r+1)r + (r+1) - \frac{1}{4}] = a_1 [(r+1)^2 - \frac{1}{4}] = 0 \end{array} \right.$$

$$\begin{array}{l} x^{n+r} \end{array} \left\{ \begin{array}{l} a_n [(n+r)(n+r-1) + (n+r) - \frac{1}{4}] + 4a_{n-2} = 0 \\ a_n [(n+r)^2 - \frac{1}{4}] + 4a_{n-2} = 0 \Rightarrow a_n = -4 \cdot \frac{1}{(n+r)^2 - \frac{1}{4}} \cdot a_{n-2} \end{array} \right.$$

$$r^2 - \frac{1}{4} = 0 \Rightarrow r_{1,2} = \pm \frac{1}{2} \quad \text{על ידי הצבה בפתרון נקבל  $a_0 \neq 0$  וכן}$$

אם  $r = \frac{1}{2}$  או  $r = -\frac{1}{2}$  נקבל פתרונות מהצורה  $y = x^{\pm \frac{1}{2}}$  ויש גם פתרונות של הסוג  $y = x^{\pm \frac{1}{2}} \ln x$ .

$$\begin{array}{l} x^{r+1} \end{array} \left\{ \begin{array}{l} a_1 [(-\frac{1}{2}+1)^2 - \frac{1}{4}] = a_1 [\frac{1}{4} - \frac{1}{4}] = 0 \Rightarrow a_1 \text{ יכול להיות שווה ל-0} \end{array} \right.$$

$$\begin{array}{l} x^{n+r} \end{array} \left\{ \begin{array}{l} a_n = -4 \cdot \frac{a_{n-2}}{(n+r)^2 - \frac{1}{4}} = -4 \cdot \frac{a_{n-2}}{n^2 + 2rn + r^2 - \frac{1}{4}} \stackrel{r = \pm \frac{1}{2}}{=} -4 \cdot \frac{a_{n-2}}{n^2 + 2rn} \stackrel{r = \pm \frac{1}{2}}{=} -4 \cdot \frac{a_{n-2}}{n^2 - n} = -4 \cdot \frac{a_{n-2}}{n(n-1)} \end{array} \right.$$

$$a_2 = \frac{-4}{2 \cdot 1} a_0 = -4 \cdot \frac{1}{2!} a_0$$

$$a_4 = \frac{(-4)}{4 \cdot 3} \cdot a_2 = \frac{(-4)(-4)}{4 \cdot 3 \cdot 2 \cdot 1} a_0 = \frac{4^2}{4!} a_0$$

$$a_6 = \frac{(-4)}{6 \cdot 5} a_4 = \frac{-4^3}{6!} a_0$$

$$a_{2m} = \frac{(-4)^m}{(2m)!} a_0$$

$$a_3 = \frac{-4}{3 \cdot 2} a_1 = \frac{-4}{3!} a_1$$

$$a_5 = \frac{4^2}{5 \cdot 4} a_3 = \frac{4^2}{5!} a_1$$

$$a_{2m+1} = \frac{(-4)^m}{(2m+1)!} a_1$$

מספר (1d) : כל המספרים הם זוגיים

לפי (1d)

$$y_1(x) = X^{-\frac{1}{2}} \left[ a_0 \sum_{n=0}^{\infty} \frac{(-4)^n}{(2n)!} X^{2n} + a_1 \sum_{n=0}^{\infty} \frac{(-4)^n}{(2n+1)!} X^{2n+1} \right]$$

הכלל



$$X^{m+1} \left| a_1 \left[ \left(\frac{1}{2} + 1\right)^2 - \frac{1}{4} \right] = 2a_1 = 0 \Rightarrow a_1 = 0 \right.$$

$r = \frac{1}{2}$  מצא

$$X^{n+r} \left| a_n = -4 \frac{a_{n-2}}{(n+r)^2 - \frac{1}{4}} = -4 \frac{a_{n-2}}{n^2 + n + \frac{1}{4} - \frac{1}{4}} = -4 \frac{a_{n-2}}{n(n+1)} \right.$$

כל המספרים הם זוגיים ולכן  $a_1 = 0$

$$a_2 = -4 \frac{a_0}{3 \cdot 2} = \frac{-4}{3!} a_0$$

$$a_4 = -4 \frac{a_2}{5 \cdot 4} = \frac{4^2}{5!} a_0$$

$$a_{2m} = \frac{(-4)^m}{(2m)!} a_0$$

$$y_2(x) = X^{\frac{1}{2}} \left[ a_0 \sum_{n=0}^{\infty} \frac{(-4)^n}{(2n+1)!} X^{2n} \right]$$



הכלל

המספרים הם זוגיים

$$y = c_1 X^{-\frac{1}{2}} \left[ \sum_{n=0}^{\infty} \frac{(-4)^n}{(2n)!} X^{2n} \right] + c_2 X^{\frac{1}{2}} \left[ \sum_{n=0}^{\infty} \frac{(-4)^n}{(2n+1)!} X^{2n} \right]$$



1.e.  $2(x^2+x^3)y'' - (x-3x^2)y' + y = 0$

$x \cdot \frac{Q(x)}{P(x)} = - \frac{(x-3x^2) \cdot x}{2(x^2+x^3)} = \frac{x^2-3x^3}{2(x^2+x^3)} = \frac{x^2}{2x^2(1+x)} - \frac{3x^3}{2x^2(1+x)} = \frac{1-3x}{2(1+x)}$   $\Rightarrow$   $x_0 = 0$   $\Rightarrow$   $x_0$   $\neq 0$   $\Rightarrow$   $\sqrt{0}$

$x^2 \cdot \frac{R(x)}{P(x)} = x^2 \cdot \frac{1}{2x^2(1+x)} = \frac{1}{2(1+x)} \Rightarrow x_0 = 0$   $\Rightarrow$   $x_0 \neq 0 \Rightarrow \sqrt{0}$

$y(x) = \sum_{n=0}^{\infty} a_n x^{n+r} \Rightarrow y' = \sum_{n=0}^{\infty} (n+r) a_n x^{n+r-1} \Rightarrow y'' = \sum_{n=0}^{\infty} (n+r)(n+r-1) a_n x^{n+r-2}$

בנקודה הזו נציב את  $x_0 = 0$

$\sum_{n=0}^{\infty} 2(n+r)(n+r-1) a_n x^{n+r} + \sum_{n=0}^{\infty} 2(n+r)(n+r-1) a_n x^{n+r+1} - \sum_{n=0}^{\infty} (n+r) a_n x^{n+r} + \sum_{n=0}^{\infty} 3(n+r) a_n x^{n+r+1} + \sum_{n=0}^{\infty} a_n x^{n+r} = 0$

$\sum_{n=0}^{\infty} 2(n+r)(n+r-1) a_n x^{n+r} + \sum_{n=0}^{\infty} a_n (n+r) [2(n+r-1)+3] x^{n+r+1} + \sum_{n=0}^{\infty} a_n (1-n-r) x^{n+r} = 0$

$\sum_{n=0}^{\infty} 2(n+r)(n+r-1) a_n x^{n+r} + \sum_{n=1}^{\infty} a_{n-1} (n+r-1) [2(n+r-2)+3] x^{n+r} + \sum_{n=0}^{\infty} a_n (1-n-r) x^{n+r} = 0$

$a_0 [2r(r-1) + (1-r)] = 0$

$n=0$   $\Rightarrow$   $a_0 \neq 0$   $\Rightarrow$   $2r^2 - 2r - r + 1 = 0 \Rightarrow 2r^2 - 3r + 1 = 0$

$2r(r-1) + 1-r = 0 \Rightarrow 2r^2 - 2r - r + 1 = 0 \Rightarrow 2r^2 - 3r + 1 = 0$

$r_{1,2} = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3 \pm 1}{4} \Rightarrow r_1 = 1, r_2 = \frac{1}{2}$

בנקודה הזו נציב את  $x_0 = 0$

$a_n [2(n+r)(n+r-1) - (n+r-1)] + (n+r-1) [2(n+r-2)+3] a_{n-1} = 0$

$a_n [(n+r-1)(2n+2r-1)] + (n+r-1) [2(n+r-2)+3] a_{n-1} = 0$

$a_n = \frac{(n+r-1)[-3-2(n+r-2)]}{(n+r-1)(2n+2r-1)} a_{n-1} = \frac{-3-2n-2r+4}{2n+2r-1} a_{n-1} = \frac{2n+2r-1}{2n+2r-1} a_{n-1} = -a_{n-1}$

$a_n = -a_{n-1}$

$r_2 = 1, r_1 = \frac{1}{2}$   $\Rightarrow$   $a_n = (-1)^n a_0$

$a_1 = -a_0$

$a_2 = -a_1 = a_0$

$a_3 = -a_2 = -a_0$

$y_1(x) = x^1 [1 + \sum_{n=1}^{\infty} (-1)^n x^n] = x \cdot \sum_{n=0}^{\infty} (-1)^n x^n = \frac{x}{1-x}$

$y_2(x) = x^{\frac{1}{2}} [\sum_{n=0}^{\infty} (-1)^n x^n] = \sqrt{x} \cdot \frac{1}{1-x}$

$y_1(x) = \frac{x}{1-x}, y_2(x) = \frac{\sqrt{x}}{1-x}$

$$x^2 y'' + xy' + (x^2 - \frac{1}{4})y = 0$$

$$J_{\frac{1}{2}} = \sqrt{\frac{x}{\pi}} \sin x$$

$$W = \begin{vmatrix} J_{\frac{1}{2}} & J_{-\frac{1}{2}} \\ J_{\frac{1}{2}}' & J_{-\frac{1}{2}}' \end{vmatrix} = J_{\frac{1}{2}} \cdot J_{-\frac{1}{2}}' - J_{-\frac{1}{2}} \cdot J_{\frac{1}{2}}' = A e^{-\int P(x) dx} = A e^{-\int \frac{1}{x} dx} = A e^{-\ln x + \ln x_0} = A \cdot x_0 \cdot x^{-1} = A^* x^{-1}$$

$$J_{\frac{1}{2}} \cdot J_{-\frac{1}{2}}' - J_{-\frac{1}{2}} \cdot J_{\frac{1}{2}}' = A^* x^{-1}$$

סה"כ קיבלנו כן

$$J_{\frac{1}{2}}' - \frac{J_{\frac{1}{2}}}{J_{\frac{1}{2}}} \cdot J_{-\frac{1}{2}} = A^* x^{-1} \cdot J_{-\frac{1}{2}}^{-1}$$



נחזק ב  $J_{\frac{1}{2}}$  ונקבל

$$U(x) = e^{\int R(x) dx} = e^{\int \frac{J_{-\frac{1}{2}}}{J_{\frac{1}{2}}} dx} = e^{-\ln J_{-\frac{1}{2}}} = (J_{-\frac{1}{2}})^{-1}$$

(אז) אר שיהיה האינטגרל

$$J_{-\frac{1}{2}} \cdot J_{\frac{1}{2}}^{-1} - J_{\frac{1}{2}} \cdot J_{-\frac{1}{2}}^{-1} = x^{-1} \cdot J_{\frac{1}{2}}^{-2} \cdot A^*$$

(כפי) בקצרה האינטגרל ונקבל

$$(J_{\frac{1}{2}}^{-1} \cdot J_{-\frac{1}{2}})' = \frac{1}{x \cdot J_{\frac{1}{2}}^2} A^*$$

$$\int (J_{\frac{1}{2}}^{-1} \cdot J_{-\frac{1}{2}})' dx = A^* \int \frac{1}{x \cdot J_{\frac{1}{2}}^2} dx = A^* \int \frac{1}{x} \cdot \frac{dx}{\frac{2}{\pi x} \sin^2 x} = \frac{\pi A^*}{2} \int \frac{1}{\sin^2 x} dx = \left( -\frac{\pi}{2} \cot x + C \right) A^*$$

$$J_{\frac{1}{2}}^{-1} \cdot J_{-\frac{1}{2}} = -\frac{\pi}{2} \cdot \frac{\cos x}{\sin x A^*} + C A^*$$

$$J_{\frac{1}{2}} = -\frac{\pi}{2} \cdot \sqrt{\frac{2}{\pi x}} \cdot \frac{\cos x}{\sin x} \cdot \sin x \cdot A^* + C_1 \sqrt{\frac{2}{\pi x}} \cdot \sin x \cdot A^*$$

$$J_{-\frac{1}{2}} = -\sqrt{\frac{\pi}{2x}} \cos x A^* + C_1 \sqrt{\frac{2}{\pi x}} \sin x \cdot A^*$$

$$\boxed{J_{-\frac{1}{2}} = -A^* \sqrt{\frac{\pi}{2x}} \cos x}$$



נחזק  $C_1 = 0$  נקבל

ובכן

$$3. \quad 4(x+1)y'' - 2xy' - y = 0 \quad x_0 = -1$$

הצורה הכללית של הפתרון

$$\lim_{x \rightarrow -1} (x+1) \cdot \frac{-2x}{4(x+1)} = \lim_{x \rightarrow -1} -\frac{1}{2}x = \frac{1}{2}$$

$$\lim_{x \rightarrow -1} (x+1)^2 \cdot \frac{(-1)}{4(x+1)} = \lim_{x \rightarrow -1} -\frac{1}{4}(x+1) = 0$$

$$y = \sum_{n=0}^{\infty} a_n (x+1)^{n+r} \Rightarrow y' = \sum_{n=0}^{\infty} a_n (n+r)(x+1)^{n+r-1} \Rightarrow y'' = \sum_{n=0}^{\infty} a_n (n+r)(n+r-1)(x+1)^{n+r-2}$$

הצורה הכללית של הפתרון

$$\sum_{n=0}^{\infty} 4a_n (n+r)(n+r-1)(x+1)^{n+r-1} - x \sum_{n=0}^{\infty} 2a_n (n+r)(x+1)^{n+r-1} - \sum_{n=0}^{\infty} a_n (x+1)^{n+r} = 0$$

$$\sum_{n=0}^{\infty} 4a_n (n+r)(n+r-1)(x+1)^{n+r-1} - \sum_{n=0}^{\infty} 2a_n (n+r)(x+1)^{n+r} + \sum_{n=0}^{\infty} 2a_n (n+r)(x+1)^{n+r-1} - \sum_{n=0}^{\infty} a_n (x+1)^{n+r} = 0$$

$$x^{-1} \cdot a_0 [4r(r-1) + 2r] + \sum_{n=0}^{\infty} 4a_{n+1} (n+r+1)(n+r)(x+1)^{n+r} - \sum_{n=0}^{\infty} 2a_n (n+r)(x+1)^{n+r} + \sum_{n=0}^{\infty} 2a_{n+1} (n+r+1)(x+1)^{n+r} - \sum_{n=0}^{\infty} a_n (x+1)^{n+r} = 0$$

$$-\sum_{n=0}^{\infty} a_n (x+1)^{n+r} = 0$$

$$x^{-1} \left| \begin{array}{l} \text{"F(r)"} \\ a_0 [4r^2 - 4r + 2r] = 0 \Rightarrow a_0 [4r^2 - 2r] = 0 \Rightarrow 2(2r^2 - r) = 0 \Rightarrow 2(r(2r-1)) = 0 \Rightarrow r_1 = 0 \\ r_2 = \frac{1}{2} \end{array} \right.$$

$$a_{n+1} [4(n+r+1)(n+r) + 2(n+r+1)] = a_n [2(n+r) + 1] \quad 0 \leq n \quad \text{BS}$$

$$a_{n+1} [2(n+r+1)(2n+2r+1)] = a_n [2n+2r+1]$$

$$a_{n+1} = \frac{(2n+2r+1)}{2(n+r+1)(2n+2r+1)} a_n = \frac{a_n}{2(n+r+1)}$$

$r_1 = 0$  7/26

$$a_1 = \frac{a_0}{2(1+0+1)} = \frac{1}{4} a_0$$

$$a_2 = \frac{a_1}{2(2+1)} = \frac{1}{6} a_1 = \frac{1}{6} \cdot \frac{1}{4} a_0 = \frac{1}{2^2 \cdot 3!} a_0$$

$$a_3 = \frac{a_2}{2(3+1)} = \frac{1}{8} a_2 = \frac{1}{8} \cdot \frac{1}{6} \cdot \frac{1}{4} a_0 = \frac{1}{2^3 \cdot 4!} a_0$$

$$\boxed{a_n = \frac{1}{2^n \cdot (n+1)!} a_0} \Rightarrow y_1(x) = (x+1)^0 a_0 \sum_{n=0}^{\infty} \frac{1}{2^n \cdot (n+1)!} (x+1)^n \Rightarrow \boxed{y_1(x) = a_0 \sum_{n=0}^{\infty} \frac{1}{2^n \cdot (n+1)!} (x+1)^n}$$

$$a_1 = \frac{a_0}{2(1+\frac{1}{2}+1)} = \frac{1}{5} a_0$$

$$a_3 = \frac{a_2}{2(2+\frac{1}{2}+1)} = \frac{1}{9!!} a_0$$

$r_2 = \frac{1}{2}$  7/26

$$a_2 = \frac{a_1}{2(2+\frac{1}{2}+1)} = \frac{1}{7} a_1 = \frac{1}{7} \cdot \frac{1}{5} a_0 = \frac{1}{2!!} a_0$$

$$\boxed{a_n = \frac{3}{(2(n+1))!!} a_0}$$

$$\boxed{y_2(x) = (x+1)^{\frac{1}{2}} a_0 \sum_{n=0}^{\infty} \frac{3}{(2(n+1))!!} (x+1)^n}$$

1/15

4. (c)  $y'' + x^2 y = 0$

$y = u \cdot x^{\frac{1}{2}}$  נבחרנו פתרון בצורתו  $y = u \cdot x^{\frac{1}{2}}$  כדי להפחית את המשוואה

$y' = u' x^{-\frac{1}{2}} - \frac{1}{2} u x^{-\frac{3}{2}}$

$y'' = u'' x^{-\frac{1}{2}} - \frac{1}{2} u' x^{-\frac{3}{2}} - \frac{1}{2} u' x^{-\frac{3}{2}} + \frac{3}{4} u x^{-\frac{5}{2}}$

$u'' x^{-\frac{1}{2}} - u' x^{-\frac{3}{2}} + (x^{\frac{3}{2}} + \frac{3}{4} x^{-\frac{5}{2}}) u = 0 \quad | \cdot x^{\frac{5}{2}}$

$x^2 u'' - x u' + (x^4 + \frac{3}{4}) u = 0$

$\left[ x \frac{(-x)}{x^2} = 1 \quad \text{כאשר } \frac{(x^4 + \frac{3}{4})}{x^2} \rightarrow \frac{3}{4}$

$u'' = \sum_{n=0}^{\infty} a_n (r+n)(r+n-1) x^{r+n-2} \in u' = \sum_{n=0}^{\infty} a_n (r+n) x^{r+n-1} \in u = \sum_{n=0}^{\infty} a_n x^{r+n}$

נכנס בתוך המשוואה

$\sum_{n=0}^{\infty} a_n (r+n)(r+n-1) x^{r+n} - \sum_{n=0}^{\infty} a_n (r+n) x^{r+n} + \sum_{n=0}^{\infty} a_n x^{r+n+4} + \sum_{n=0}^{\infty} \frac{3}{4} a_n x^{r+n} = 0$

נציב בתוך המשוואה

$\sum_{n=0}^{\infty} a_n (r+n)(r+n-1) x^{r+n} - \sum_{n=0}^{\infty} a_n (r+n) x^{r+n} + \sum_{n=4}^{\infty} a_{n-4} x^{r+n} + \sum_{n=0}^{\infty} \frac{3}{4} a_n x^{r+n} = 0$

$n=0 \quad a_0 [ (r-1)r - r + \frac{3}{4} ] = 0$

$r^2 - r - r + \frac{3}{4} = 0$

$r^2 - 2r + \frac{3}{4} = 0 \Rightarrow r_{1,2} = \frac{2 \pm \sqrt{4-3}}{2} = \begin{cases} r_1 = \frac{3}{2} \\ r_2 = \frac{1}{2} \end{cases}$

$n=1 \quad a_1 [ (r+1)r - (r+1) + \frac{3}{4} ] = 0$

$a_1 [ r^2 + r - r - 1 + \frac{3}{4} ] = 0$

$a_1 [ r^2 - \frac{1}{4} ] = 0 \Rightarrow a_1 (r + \frac{1}{2})(r - \frac{1}{2}) = 0 \Rightarrow$

$a_1 = 0$  כי  $r_1, r_2$  אינם שווים

$n=2 \quad a_2 [ (r+2)(r+1) - (r+2) + \frac{3}{4} ] = 0$

$a_2 [ r^2 + r + 2r + 1 - r - 2 + \frac{3}{4} ] = 0$

$a_2 [ (r + \frac{1}{2})(r + \frac{3}{2}) ] = 0$

$a_2 = 0$  כי  $r_1, r_2$  אינם שווים

$n=3 \quad a_3 [ (r+3)(r+2) - (r+3) + \frac{3}{4} ] = 0$

$a_3 [ r^2 + 2r + 3r + 6 - r - 3 + \frac{3}{4} ] = 0$

$a_3 [ r^2 + 4r - \frac{11}{4} ] = 0$

$a_3 = 0$  כי  $r_1, r_2$  אינם שווים

$n \geq 4 \quad a_n [ (r+n)(r+n-1) - (r+n) + \frac{3}{4} ] + a_{n-4} = 0$

$a_n = - \frac{(r+n)(r+n-1) - (r+n) + \frac{3}{4}}{(r+n)(r+n-1) - (r+n) + \frac{3}{4}} a_{n-4}$

$a_n = - \frac{1}{(r+n-\frac{1}{2})(r+n-\frac{3}{2})} a_{n-4}$

$r_2 = \frac{1}{2}$  (צורת הפתרון הכללי)  $\forall n \geq 4$   $a_n = - \frac{1}{(r+n-\frac{1}{2})(r+n-\frac{3}{2})} a_{n-4}$

$a_n = - \frac{1}{n(n-1)} a_{n-4}$

$a_4 = - \frac{1}{4 \cdot 3} a_0$

$a_8 = - \frac{1}{8 \cdot 7} a_4 = \frac{1}{8 \cdot 7} \cdot \frac{1}{4 \cdot 3} a_0$

$a_{4n} = (-1)^n \frac{(n-2)(n-3) \dots (n-6)(n-7)}{n!} a_0$

מקבלים  $a_1, a_2, a_3$  קיבלנו  $4n+3, 4n+2, 4n+1$  סדרה חשבונית,  $4n+1$  סדרה חשבונית,  $0$  סדרה חשבונית

$$y_1(x) = x^{\frac{1}{2}} a_0 \left[ 1 + \sum_{n=1}^{\infty} \frac{(-1)^n [(n-2)(n-3)] \dots 2 \cdot 1}{n!} x^{4n} \right]$$

סדרה קיבלנו  $\frac{1}{2}$

(ק)  $r = \frac{3}{2}$  נכנס

$$a_n = -\frac{1}{n(n+1)} a_{n-4}$$

$$a_4 = -\frac{1}{5 \cdot 4} a_0$$

$$a_8 = -\frac{1}{9 \cdot 8} a_4 = \frac{1}{9 \cdot 8} \cdot \frac{1}{5 \cdot 4} a_0$$

$$a_{12} = -\frac{1}{13 \cdot 12} a_8 = -\frac{1}{13 \cdot 12} \cdot \frac{1}{9 \cdot 8} \cdot \frac{1}{5 \cdot 4} a_0$$

$$a_{4n} = \frac{(-1)^n [(n-1)(n-2)] \dots 3 \cdot 2}{(n+1)!} a_0$$

מקבלים  $a_1, a_2, a_3$  קיבלנו  $4n+3, 4n+2, 4n+1$  סדרה חשבונית,  $4n+1$  סדרה חשבונית,  $0$  סדרה חשבונית

סדרה קיבלנו  $\frac{3}{2}$

$$y_2(x) = x^{\frac{3}{2}} a_0 \left[ 1 + \sum_{n=1}^{\infty} \frac{(-1)^n [(n-1)(n-2)] \dots 3 \cdot 2}{(n+1)!} x^{4n} \right]$$

סדרה קיבלנו  $\frac{3}{2}$

סדרה קיבלנו  $\frac{3}{2}$

$$u(x) = C_1 x^{\frac{1}{2}} \left[ 1 + \sum_{n=1}^{\infty} \frac{(-1)^n [(n-2)(n-3)] \dots 2 \cdot 1}{n!} x^{4n} \right] + C_2 x^{\frac{3}{2}} \left[ 1 + \sum_{n=1}^{\infty} \frac{(-1)^n [(n-1)(n-2)] \dots 3 \cdot 2}{(n+1)!} x^{4n} \right]$$

$$y(x) = C_1 \left[ 1 + \sum_{n=1}^{\infty} \frac{(-1)^n [(n-2)(n-3)] \dots 2 \cdot 1}{n!} x^{4n} \right] + C_2 x \left[ 1 + \sum_{n=1}^{\infty} \frac{(-1)^n [(n-1)(n-2)] \dots 3 \cdot 2}{(n+1)!} x^{4n} \right]$$

$$5. \quad y'' + 2y' - iy = 0$$

$$\text{בפני 13) } y'' = r^2 e^{rx} \Leftarrow y' = r e^{rx} \Leftarrow y = e^{rx} \quad \text{מבנה הפתרון}$$

$$r^2 e^{rx} + 2r e^{rx} - i e^{rx} = 0 \quad || e^{rx} \neq 0$$

$$r^2 + 2r - i = 0$$

$$r_{1,2} = \frac{-2 \pm \sqrt{4 + 4i}}{2} = \frac{-2 \pm 2\sqrt{1+i}}{2} = -1 \pm \sqrt{1+i}$$

$$r = \sqrt{1+i} = \sqrt{2}$$

$$\text{מבנה הפתרון } \sqrt{1+i} \quad \text{מבנה הפתרון}$$

$$\cos \theta = \frac{1}{\sqrt{2}} \quad ; \quad \sin \theta = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \theta = \frac{\pi}{4}$$

$$\sqrt{1+i} = \sqrt{2} \left( \cos \frac{\pi}{8} + i \sin \frac{\pi}{8} \right) = 1.098 + i0.455$$

$$r_{1,2} = -1 \pm (1.098 + i0.455) \quad \left\{ \begin{array}{l} r_1 = -2.098 - i0.455 \\ r_2 = 0.098 + i0.455 \end{array} \right.$$

$$y(x) = c_1 e^{(-2.098 - i0.455)x} + c_2 e^{(0.098 + i0.455)x}$$

כאן, הפתרון

$$y(0) = c_1 e^0 + c_2 e^0 = c_1 + c_2 = 0 \Rightarrow c_1 = -c_2$$

כאן, הפתרון

$$y'(0) = c_1 (-1 - \sqrt{1+i}) e^0 + c_2 (-1 + \sqrt{1+i}) e^0 = 0$$

$$c_1 (-1 - \sqrt{1+i}) - c_1 (-1 + \sqrt{1+i}) = 0$$

$$-c_1 - c_1 \sqrt{1+i} + c_1 - c_1 \sqrt{1+i} = 0$$

$$c_1 = -\frac{1}{2\sqrt{1+i}} = -\frac{1}{2(1.098 + i0.455i)} \Rightarrow c_2 = \frac{1}{2(1.098 + i0.455i)}$$

$$y(x) = -\frac{1}{2\sqrt{1+i}} e^{(-2.098 - i0.455)x} + \frac{1}{2(1.098 + i0.455i)} e^{(0.098 + i0.455)x}$$

כאן, הפתרון