

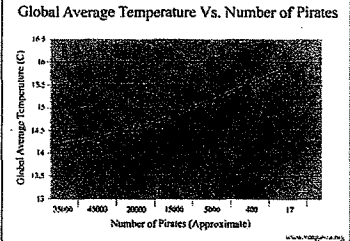
## Animation

ליאור שפירא  
גרפיקה ממוחשבת  
סמסטר ב' תשס"ט

מבוסס על שקפים של תומס פוקהאוסר, פרינסטון

### סקר הוראה

- סקר סקר סקר
- נמשך 31.5 – 20.6
- חשוב!!
- מאוד!!



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### הצצה לשבוע הבא...

- רן גל יספר לנו על Wires
- גיל הופר יספר לנו על "Coordinates for Instant Image Cloning", אלטרנטיבה ל-Poisson
- ג'קי יספר לנו על אנמציה של דמויות אמטיות ותמונות תנועות

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### Syllabus

- I. Image processing
- II. Modeling
- III. Rendering
- IV. Animation







Image Processing  
(Rony Coleman, CS226, Fall09)



Rendering  
(Michael Bosock, CS426, Fall09)




Modeling  
(Donita Zarin, CalTech)



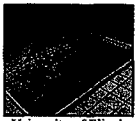
Animation  
(Angel Plets 1)

### Computer Animation

- What is animation?
  - Make objects change over time according to scripted actions
- What is simulation?
  - Predict how objects change over time according to physical laws





Pixar



University of Illinois

### Computer Animation





Pixar

א אויבויף ייהא בעלם והגעטען ט' תרה  
כוסר פ'וקים בעלם

### Outline

- > Keyframe animation
- Adding inverse kinematics
- Adding dynamics

Lior '09

### Keyframe Animation

- Define character poses at specific time steps called "keyframes"

Lasseter '87

### Keyframe Animation

- Interpolate variables describing keyframes to determine poses for character "in-between"

Lasseter '87

### Example: 2-Link Structure

- Two links connected by rotational joints

(0,0)

כאשר יש לנו מרחב, אנו יכולים לייצר תווך פרימיטיב וזאת למחשב להשלים את התנועה. ההשלמה הזו היא יחסית וזה לא תמיד פשוט.

ניתן להגדיר keyframe באמצעות הזנת הנתונים. זה כק (x) מיקום של X (y).

### Forward Kinematics

- Animator specifies joint angles:  $u_1$  and  $u_2$
- Computer finds positions of end-effector: X

(0,0)

$$X = (l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2), l_1 \sin \theta_1 + l_2 \sin(\theta_1 + \theta_2))$$

מבחינת חישובים זה פשוט.

### Forward Kinematics

- Joint motions can be specified by spline curves

(0,0)

ניתן להגדיר את התנועה של המפרקים באמצעות spline.

### Keyframe Animation

- Inbetweening:
  - Linear interpolation - usually not enough continuity

Linear interpolation

H&B Figure 16.16

### Keyframe Animation

- Inbetweening:
  - Spline interpolation - maybe good enough

Key Frame  $k$

In-Between

Key Frame  $k+1$

Key Frame  $k+2$

H&B Figure 16.11

האנטיגו, אנטיגו או הקובוקר. (כזו) אידור או התנועה  
 הרגויה (הקו האדום). האנטיגו או הקובוקר של הקובוקר  
 על הרוב לניסיון (13 שטח אחר) האנטיגו או הקובוקר של הקובוקר  
 אלא ונדרים "קובוקר חזק" אנטיגו או הקובוקר של הקובוקר והיו אנטיגו  
 חזק.

### Keyframe Animation

- Inbetweening:
  - Cubic spline interpolation - maybe good enough
  - » May not follow physical laws

Lasseter '87

### Keyframe Animation

- Inbetweening:
  - Cubic spline interpolation - maybe good enough
  - » May not follow physical laws

Lasseter '87

כזו אנטיגו ונדרים או הקובוקר  
 נכנסים או הקובוקר חזק.

### Example: Walk Cycle

- Articulated figure:
  - Hip
  - Upper leg
  - Knee
  - Lower leg
  - Ankle
  - Foot

Upper leg (hip rot)

Hip rotate

Lower leg (knee rot)

Hip rotate + knee rot

Foot (ankle rot)

Watt & Watt

### Example: Walk Cycle

- Hip joint orientation:
  - Graph showing hip joint orientation over a walk cycle.

45°

-35°

1 2 3 4 5

Watt & Watt

אם היינו חולבים על שיני דק  
 בזווית של ה Hip, (ההצבה הימית)  
 אנטיגו (נתיב).

### Example: Walk Cycle

- Knee joint orientation:

Watt & Watt

### Example: Walk Cycle

- Ankle joint orientation:

Watt & Watt

### Outline

- Keyframe animation
- Adding inverse kinematics
- Adding dynamics

בידור (כאשר יש להם מודל של האובייקט) מוסיפים קליפ אנטיאל

### Example: 2-Link Structure

- What if animator knows position of "end-effector"

מקום המבוקש ממשלב ומקבל את המיקום  
 כך ש-X יהיה במקום המבוקש. זה  
 מאזן פשוט.

### Inverse Kinematics

- Animator specifies end-effector positions: X
- Computer finds joint angles:  $\theta_1$  and  $\theta_2$ :

$$\theta_2 = \cos^{-1} \left( \frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1 l_2} \right)$$

$$\theta_1 = \frac{-(l_2 \sin(\theta_2)x + (l_1 + l_2 \cos(\theta_2))y)}{(l_2 \sin(\theta_2))y + (l_1 + l_2 \cos(\theta_2))x}$$

ה'פנים' של ה'גולסה'...

### Inverse Kinematics

- End-effector positions can be specified by spline curves

### Inverse Kinematics

- Problem for more complex structures
  - System of equations is usually under-defined
  - Multiple solutions

Three unknowns:  $u_1, u_2, u_3$   
Two equations:  $x, y$

עכשיו קורה שהמיקום שלנו נחשב פשוט  
והזוויות נחשבות מהמשוואות (הזוויות) של  
המבנה (ולכן זהו בעיה מרובת פתרונות)

### Inverse Kinematics

- Solution for more complex structures:
  - Find best solution (e.g., minimize energy in motion)
  - Non-linear optimization

אנחנו אין רק פתרון אחד, אלא אולי  
עשרות או מאות הפתרונות וביניהם  
הנמוכה או זהו המצויינות  
היא יכולה להתבסס על מינימום אנרגיה

### Outline

- Keyframe animation
- Adding inverse kinematics
- Adding dynamics

אנחנו יכולים להחזיק את המבנה  
עבורה שיש לה על המפה

### Dynamics

- Simulation of physics insures realism of motion

Lasseier '87

פתרון  
אם המבנה  
יש המבנה  
key frame  
הוא זה  
הוא המבנה

כך אני מבין שזהו אינטליגנטי (ראויות)  
המבנה (המבנה) המיושם על המפה

### Spacetime Constraints

- Animator specifies constraints:
  - What the character's physical structure is
    - e.g., articulated figure
  - What the character has to do (keyframes)
    - e.g., jump from here to there within time  $t$
  - What other physical structures are present
    - e.g., floor to push off and land
  - How the motion should be performed
    - e.g., minimize energy

המבנה ככה, המבנה יושב וזיגור  
המבנה זה המבנה המיושם המבנה

### Spacetime Constraints

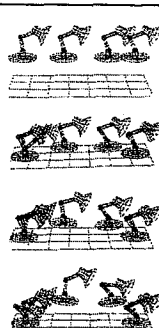
- Computer finds the "best" physical motion satisfying constraints
- Example: particle with jet propulsion
  - $x(t)$  is position of particle at time  $t$
  - $f(t)$  is force of jet propulsion at time  $t$
  - Particle's equation of motion is:
 
$$mx'' - f - mg = 0$$
  - Suppose we want to move from  $a$  to  $b$  within  $t_0$  to  $t_1$  with minimum jet fuel:
 
$$\text{Minimize } \int_{t_0}^{t_1} |f(t)|^2 dt \text{ subject to } x(t_0)=a \text{ and } x(t_1)=b$$

Witkin & Kass '88

המבנה יש את זה המבנה המיושם  
המבנה זה המבנה המיושם המבנה  
המבנה

### Spacetime Constraints

- Solve with iterative optimization methods



Witkin & Kass '88

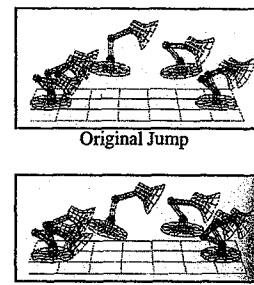
כדי לנתח את הפעולה הממוקדת ביותר, נחלק את המטרה למסלול, תחילתו עד שנגיע לתחילת המטרה.

### Spacetime Constraints

- Advantages:
  - Free animator from having to specify details of physically realistic motion with spline curves
  - Easy to vary motions due to new parameters and/or new constraints
- Challenges:
  - Specifying constraints and objective functions
  - Avoiding local minima during optimization

### Spacetime Constraints

- Adapting motion:



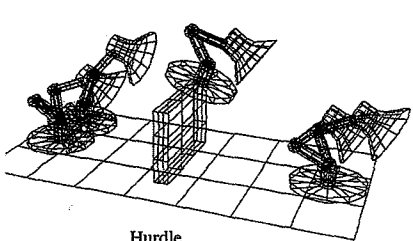
Original Jump

Heavier Base

Witkin & Kass '88

### Spacetime Constraints

- Adapting motion:



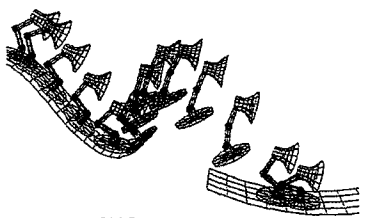
Hurdle

Witkin & Kass '88

מגדירים את המסלול של ה'2' וה'1' על ידי יצירת מישור חדש של הכתם של המטרה.

### Spacetime Constraints

- Adapting motion:




Ski Jump

Witkin & Kass '88

### Spacetime Constraints

- Advantages:
  - Free animator from having to specify details of physically realistic motion with spline curves
  - Easy to vary motions due to new parameters and/or new constraints
- Challenges:
  - Specifying constraints and objective functions
  - Avoiding local minima during optimization

### Example: Manipulation of Sims.



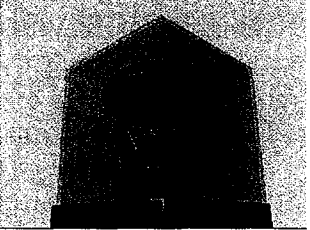
Interactive Manipulation of Rigid Body Simulations.  
Popovic et al Siggraph 2000.

Popovic

### Summary

- Keyframe animation
  - Poses specified at key times
  - In-betweening to fill in the rest
- Incorporating inverse kinematics
  - Makes keyframes easier to specify
- Incorporating dynamics
  - Makes animation easier to adapt

# Simulation



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### Simulation

- Dynamics
  - Considers underlying forces
  - Compute motion from initial conditions and physics
- Kinematics
  - Considers only motion
  - Determined by positions, velocities, accelerations

### Dynamics

Passive--no muscles or motors

user → initial conditions → model (numerical integrator) → state → graphics

particle systems  
leaves  
water spray  
clothing

Active--internal source of energy

user → desired behavior → control → forces and torques → model (numerical integrator) → state → graphics

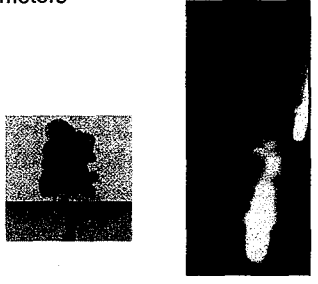
running human  
trotting dog  
swimming fish

Hodgins

*Handwritten notes:* מ'ס' (כ) 18, 16 כח מלן, ארן מלג, 208

### Passive Dynamics


- No muscles or motors
  - Smoke
  - Water
  - Cloth
  - Fire
  - Fireworks
  - Dice



McAllister

### Passive Dynamics

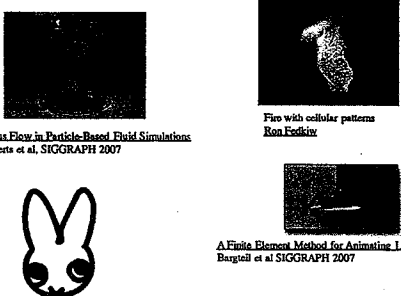
- Physical laws
  - Newton's laws
  - Hook's law
  - Etc.
- Physical phenomena
  - Gravity
  - Momentum
  - Friction
  - Collisions
  - Elasticity
  - Fracture



McAllister

התדרכת מרובת נחתון חלקי קטנים שמה  
וירגיש את השלם

### Fun with Bunny



Porous Flow in Particle-Based Fluid Simulations  
Lenearts et al. SIGGRAPH 2007


Fun with cellular patterns  
Ron Fodtkiv

A Finite Element Method for Animating Large Viscoplastic Flow  
Bargteil et al SIGGRAPH 2007

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### Particle Systems

- A particle is a point mass
  - Mass
  - Position
  - Velocity
  - Forces
  - Color
  - Lifetime
- Use lots of particles to model complex phenomena
  - Keep array of particles
  - Newton's laws




### Our Particle

```
enum ParticleType { Create, Update };

struct Vector2d{
    double x;
    double y;
};

struct Particle {
    Vector2d Pos; //Position of the particle
    Vector2d Vel; //Velocity of the particle
    int age; //Current age of the particle
    int LifeSpan; //Age after which the particle dies
    int color;
    int size;
};
```



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### Particle Systems

- For each frame:
  - Create new particles and assign attributes
  - Delete any expired particles
  - Update particles based on attributes and physics
  - Render particles

### Our Particle

```
void Init(long num_part, long num_forces, Vector2d Forces[],
    ParticleType part_type, Vector2d vel, Vector2d pos1, Vector2d
    pos2, int lifespan, int color, int size){

    Particles = new Particle[num_part];
    ParticleNum=num_part;

    for(int i=0; i<num_forces; i++){
        TotForce.x+=Forces[i].x;
        TotForce.y+=Forces[i].y;
    }

    Particle_Type=part_type;
    Vel=vel;
    Pos1=pos1;
    Pos2=pos2;
    LifeSpan=lifespan;
    Color=color;
    Size=size;
    randomize();
    InitParticles();
}
```

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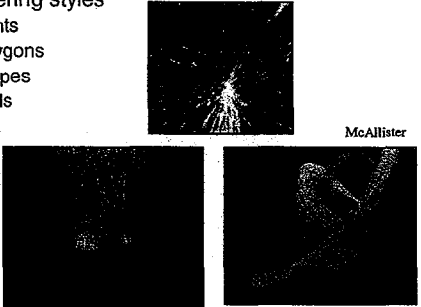
התקנים של התוכנית בין פריט לפריט  
שיהיה שונים או שיהיה זהה בתפקיד  
עליו התקנים. הכללה עם הלימו העליון  
עבודה יותר נוחה





### Rendering Particles

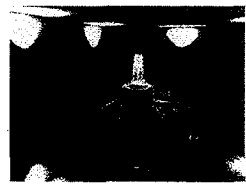
- Rendering styles
  - Points
  - Polygons
  - Shapes
  - Trails
  - etc.



McAllister

### Particle Systems

- For each frame:
  - Create new particles and assign attributes
  - Delete any expired particles
  - Update particles based on attributes and physics
  - Render particles



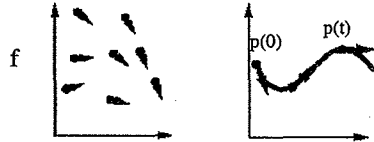
McAllister

### Equations of Motion

- Newton's Law for a point mass
  - $f = ma$
- Computing particle motion requires solving second-order differential equation
 
$$\ddot{x} = \frac{f(x, \dot{x}, t)}{m}$$
- Add variable  $v$  to form coupled first-order differential equations
 
$$\begin{cases} \dot{x} = v \\ \dot{v} = \frac{f}{m} \end{cases}$$

### Solving the Equations of Motion

- Initial value problem
  - Know  $p(0), v(0), a(0)$
  - Can compute force at any time and position
  - Compute  $p(t)$  by forward integration



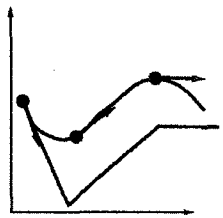
Hodgins

האם נוסף 100% → תאורה זה (בלתי שניה) של חלקים (מסתבר אולי חלקו פ'צ'קרה)

קני חלום הם על התחלקים יש מלא עפ"י מאמקלה חשיתא חסד ראלין

### Solving the Equations of Motion

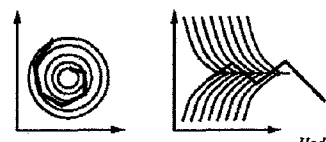
- Euler integration
  - $p(t+\Delta t) = p(t) + \Delta t v(t)$
  - $v(t+\Delta t) = v(t) + \Delta t f(x(t), t)/m$



Hodgins

### Solving the Equations of Motion

- Euler integration
  - $p(t+\Delta t) = p(t) + \Delta t v(t)$
  - $v(t+\Delta t) = v(t) + \Delta t f(p(t), t)/m$
- Problem:
  - Accuracy decreases as  $\Delta t$  gets bigger



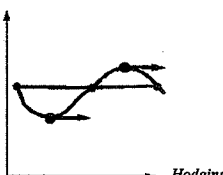
Hodgins

באל שבתים על מנתם במליון קיימת שגיאה שלבד, (המיו סטטיסטי), היתרון - מאלק חביר, הסברין א אילד חים חסד ראלין

חיים או מלמטים בקרב א אילד אא בקיוב חסד שני א רבלי

### Solving the Equations of Motion

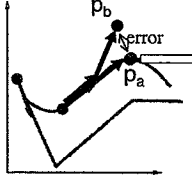
- Midpoint method (2<sup>nd</sup> order Runge-Kutta)
  - Compute an Euler step
  - Evaluate  $f$  at the midpoint
  - Take an Euler step using midpoint force
    - $v(t+\Delta t) = v(t) + \Delta t f(p(t) + 0.5 \Delta t v(t), t)$



מתי ממוקד (ממוקד) אנו מנסים. אחרת לא, בקצב ממוקד נקרא מסדר רביעי.

### Solving the Equations of Motion

- Adapting step size
  - Compute  $p_a$  by taking one step of size  $h$
  - Compute  $p_b$  by taking 2 steps of size  $h/2$
  - Error =  $|p_a - p_b|$
  - Multiply step size by factor (constant/error)

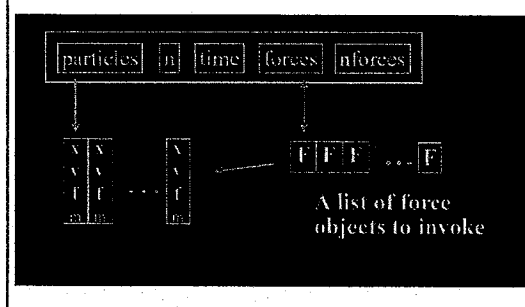


### Particle System Forces

- Force fields
  - Gravity, wind, pressure
- Viscosity/damping
  - Liquids, drag
- Collisions
  - Environment
  - Other particles
- Other particles
  - Springs between neighboring particles (mesh)
  - Useful for cloth


כחית מופיעים על חלקיקים, גם בין חלקיקים ויש 'קטגוריות' זה מוזכר חומר הקובע את.

### Particle System Forces

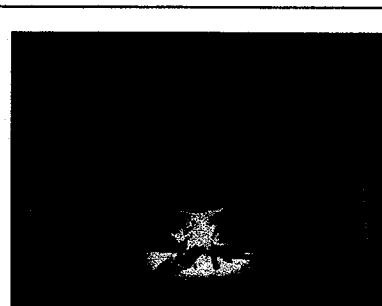


אם כל הכוליות סכומים בקו אחד, זה פונקציה  $f$  (גם נק' זמן בקו) חלקים במל, אזו כחית מופיעים גם.

### Example: Gravity

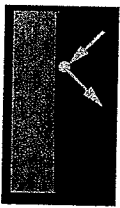


### Example: Fire



### Example: Bouncing Off Wall

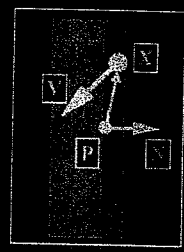
- Requires
  - Collision detection
  - Collision response (dynamic forces)



*Witkin*

### Example: Bouncing Off Wall

#### Collision Detection



$$(X - P) \cdot N < \epsilon$$

$$N \cdot V < 0$$

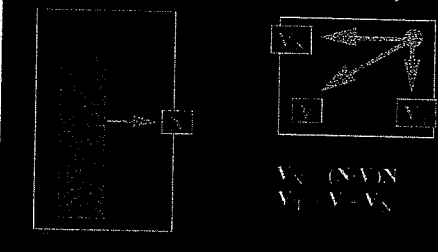
- Within  $\epsilon$  of the wall.
- Heading in.

*Witkin*

התנגשות עם סכר - קי אבנך התנגש  
 עם קיר, התקרה כבר הפעילה כח הפך  
 עקר הפעולה

### Example: Bouncing Off Wall

#### Normal and Tangential Components



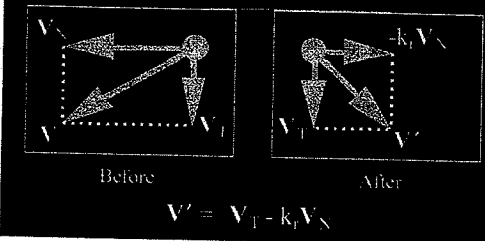
$$V_N = (V \cdot N)$$

$$V_T = V - V_N$$

*Witkin*

### Example: Bouncing Off Wall

#### Collision Response



Before After

$$V' = V_T - k_r V_N$$

*Witkin*

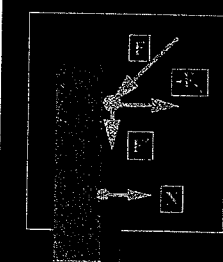
### Example: Bouncing Off Wall

#### Contact Force

$$F' = F_1$$

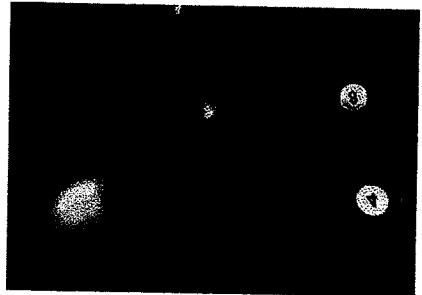
The wall pushes back, cancelling the normal component of  $F$ .

(An example of a constraint force.)



*Witkin*

### Example: Bouncing Off Particles



### Advancing our Particles

```

void Run() {
    while (!Quit()) {
        Delay(10);
        ClearDevice();
        //Delay(10);
        for (int i=0; i<ParticleNum; i++) {
            if (Particle[i].Age > 3000)
                continue;
            Vec3 pos = Particle[i].Pos;
            Vec3 vel = Particle[i].Vel;
            Vec3 acc = Particle[i].Acc;
            //Update (int) Particle[i].Age, Vec3 pos, Vec3 vel, Vec3 acc;
            Particle[i].Pos = pos + vel + acc;
            Particle[i].Vel = vel + acc;
            Particle[i].Acc = acc;
            //Draw Particle
            DrawParticle(Particle[i].Pos, Particle[i].Vel);
        }
    }
}
    
```

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### Example: Cloth

- Spring-mass mesh
- Hooke's law

$$f = -k_s(|d| - s) \frac{d}{|d|}$$

f = force  
 k<sub>s</sub> = spring constant  
 d = p - q  
 s = resting length

Hodgins

### Example: Cloth

- Spring-mass mesh

Hodgins

### Example: Cloth

Animating Developable Surfaces

Simulating Knitted Cloth at the Yarn Level

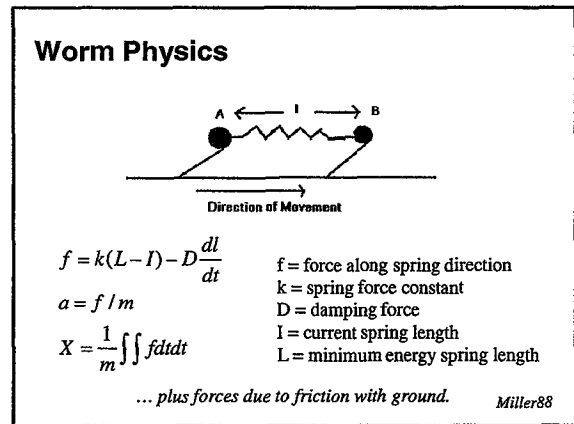
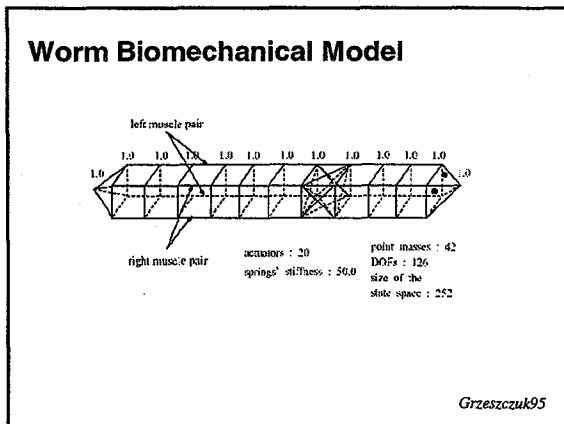
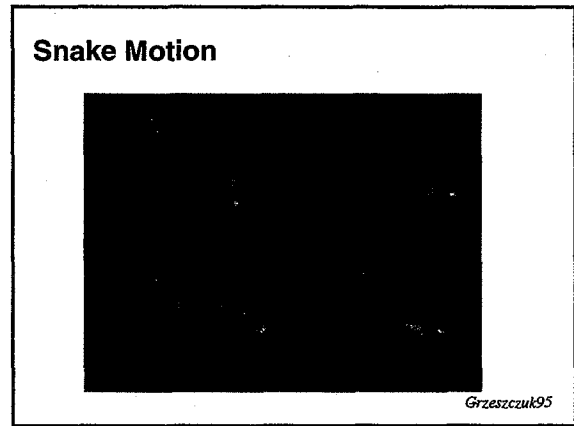
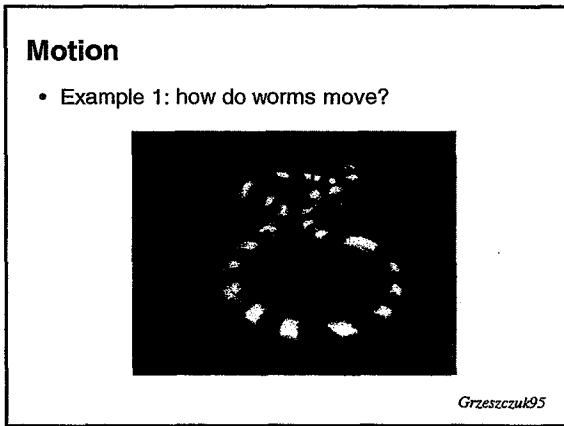
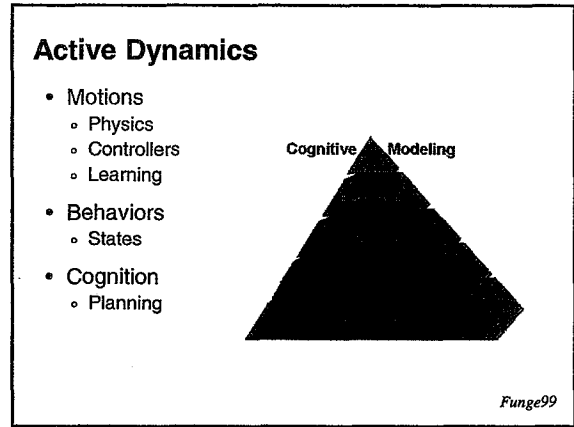
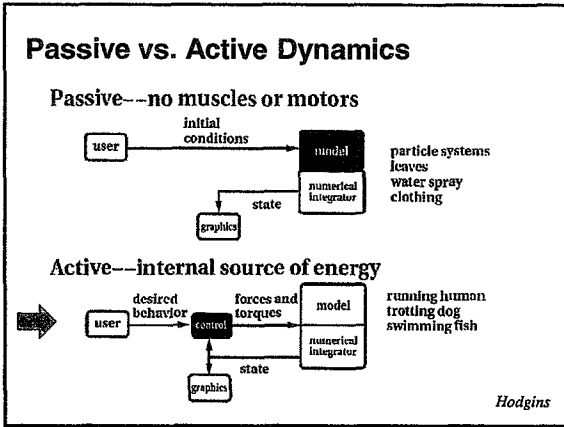
### Example: Flocks & Herds

Greg M. Johnson Copyright 2000. All Rights Reserved.

### Summary

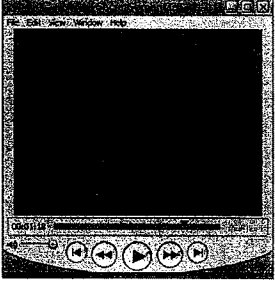
- Particle systems
  - Lots of particles
  - Simple physics
- Interesting behaviors
  - Waterfalls
  - Smoke
  - Cloth
  - Flocks
- Solving motion equations
  - Simplest method is Euler integration
  - Better to use adaptive step sizes

יש קבוצה של חלקיקים שכל אחד מהם  
 (אנשים, דגים, עופות)  
 הם חלקיקים המזיזים חלקים אחרים  
 בחלקיקים אחרים.  
 כל חלקיקים אלו הם חלקים  
 של...

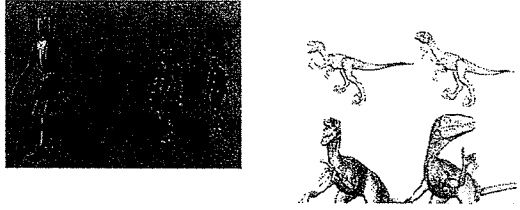


מגורים אף העל כמו מדינה  
 קפלים (כמה) עכבר לתהלים  
 וכו' (ממחז)

### Her Majesty's Secret Serpent




Miller89



Time permitting...

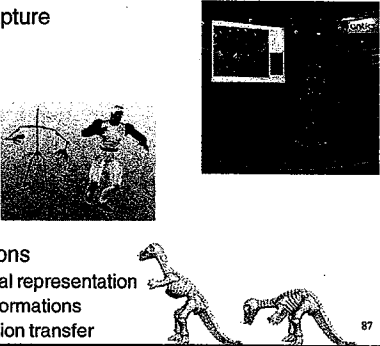
### OTHER TOPICS



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### Other topics in Animation

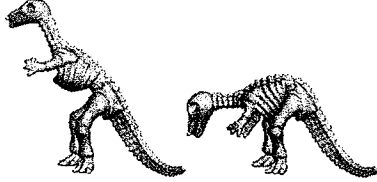
- Motion Capture
- Skinning
- Deformations
  - Differential representation
  - Cage deformations
  - Deformation transfer



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### What do we expect from surface deformation?

- Smooth effect on the large scale
- As-rigid-as-possible effect on the small scale (preserves details)



### Several approaches


- FFD (space deformation)
  - Lattice-based (Sederberg & Parry 86, Coquillart 90, ...)
  - Curve/handle-based (Singh & Fiume 98, Botsch et al. 05, ...)
  - Cage-based (Ju et al. 05, Joshi et al. 07, Kopf et al. 07)
- Pros:
  - efficiency almost independent of the surface resolution
  - possible reuse
- Cons:
  - space warp, so can't precisely control surface properties



Images taken from (Sederberg and Parry 86) and Liu et al. 02

### Several approaches

- Surface-based approaches
  - Multiresolution modeling (Zorin et al. 97, Kobbelt et al. 98, Lee 98, Guskov et al. 99, Botsch and Kobbelt 04, ...)
  - Differential coordinates – linear optimization (Lipman et al. 04, Sorkine et al. 04, Yu et al. 04, Lipman et al. 05, Zayer et al. 05, Botsch et al. 06, Fu et al. 06, ...)
  - Non-linear global optimization approaches (Kraevoy & Sheffer 04, Sumner et al. 05, Hunag et al. 06, Au et al. 06, Botsch et al. 06, Shi et al. 07, ...)



Images taken from PIVA, Botsch et al. 06

הוא מראה motion capture, חייב להיות  
 שיש קישורים אל התנועה והוא קיבץ עזרים  
 חייב לקבץ את כל הפונקציות והוא כן הוא  
 יתקן משהו/הקטן אותו ככה. (אם אפשר להוסיף כמה  
 שרשומות ככה  
 עדיין משהו  
 חזק (אולי...)

ב' שני בקומה ה'ב'א'א' י'א'א'  
 ע'ק'ב'ס' א'ה'ו' ח'ו'ג' ש'ל' ה'א'ב'י'ק'ס' ה'ק'י'ב'ר' ע'פ'י'א'ט'א'צ'ר'  
 ש' פ'ר'מ'א'ט'ר'ו'ן'

### Surface-based approaches

- Pros:
  - direct interaction with the surface
  - control over surface properties
- Cons:
  - linear optimization suffers from artifacts (e.g. translation insensitivity)
  - non-linear optimization is more expensive and non-trivial to implement

Sorkine et al 2004

### LAPLACIAN SURFACE EDITING

<http://www.cs.nyu.edu/~sorkine/ProjectPages/Editing/ee.html>

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### Differential Coordinates

- Differential coordinates are defined by the discrete Laplacian operator:
 
$$\delta_i = L(v_i) = v_i - \frac{1}{d_i} \sum_{j \in N(i)} v_j$$
- For highly irregular meshes: cotangent weights [Desbrun et al. 99]

### Why differential coordinates?

- They represent the local detail / local shape description
  - The direction approximates the normal
  - The size approximates the mean curvature

$$\delta_i = \frac{1}{d_i} \sum_{j \in N(i)} (v_i - v_j)$$

$$\lim_{len(\gamma) \rightarrow 0} \frac{1}{len(\gamma)} \int_{\gamma} (v_i - v) ds = H(v_i) n_i$$

### Why differential coordinates?

- Local detail representation – enables detail preservation through various modeling tasks
- Representation with sparse matrices
- Efficient linear surface reconstruction

### Overall framework

- Compute differential representation
 
$$\Delta = L(V)$$
- Pose modeling constraints
 
$$v'_i = u_i, \quad i \in C$$
- Reconstruct the surface – in least-squares sense
 
$$\tilde{V}' = \arg \min_{V'} \left( \|L(V') - \Delta\|^2 + \sum_{i \in C} \|v'_i - u_i\|^2 \right)$$

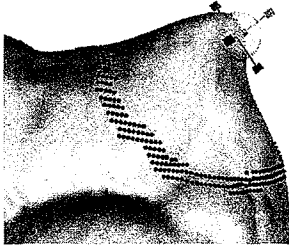
המשטח מורכב מנקודות וקווי קשר ביניהן. כל נקודה היא ממוצע של נקודות השכנות לה. המטרה היא לשחזר את המשטח בצורה הטובה ביותר.

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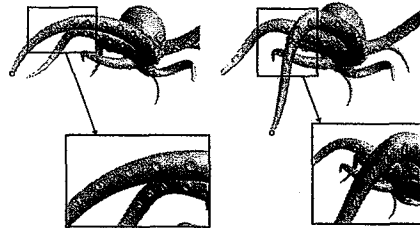
**Overall framework**

- ROI is bounded by a belt (static anchors)
- Manipulation through handle(s)



**Problem: invariance to transformations**

- The basic Laplacian operator is translation-invariant, but not rotation- and scale-invariant
- Reconstruction attempts to preserve the original global orientation of the details



**Implicit definition of transformations**

- The idea: solve for local transformations AND the edited surface simultaneously!

$$\tilde{V}' = \arg \min_{V'} \left( \sum_{i=1}^n \|L(v'_i) - T_i(\delta_i)\|^2 + \sum_{j \in C} \|v'_j - u_j\|^2 \right)$$

Transformation of the local frame

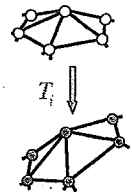
**Defining the transformations  $T_i$**

$$\tilde{V}' = \arg \min_{V'} \left( \sum_{i=1}^n \|L(v'_i) - T_i(\delta_i)\|^2 + \sum_{j \in C} \|v'_j - u_j\|^2 \right)$$

- How to formulate  $T_i$ ?
  - Based on the local (1-ring) neighborhood
  - Linear dependence on the unknown  $v'_i$

Members of the 1-ring of  $i$ -th vertex

$$\begin{cases} v'_k = T_i v_k \\ v'_l = T_i v_l \\ \vdots \\ v'_i = T_i v_i \end{cases}$$



**Defining the transformations  $T_i$**

- First attempt: define  $T_i$  simply by solving

$$T_i = \arg \min_{T_i} \sum_{j=1}^k \|v'_j - T_i v_{i_j}\|^2$$

$$\left( T_i \right) = \left( \begin{array}{c|c|c|c|c} | & | & & & | \\ v'_1 & v'_2 & \dots & \dots & v'_k \\ | & | & & & | \end{array} \right) \left( \begin{array}{c|c|c|c|c} | & | & & & | \\ v_k & v_k & & & v_k \\ | & | & & & | \end{array} \right)^*$$

**Defining the transformations  $T_i$**

- Plug the expressions for  $T_i$  into the least-squares reconstruction formula:

$$\tilde{V}' = \arg \min_{V'} \left( \sum_{i=1}^n \|L(v'_i) - T_i \delta_i\|^2 + \sum_{j \in C} \|v'_j - u_j\|^2 \right)$$

Linear combination of the unknown  $v'_i$

### Constraining $T_i$

- Trivial solution for  $T_i$  will result in membrane surface reconstruction
- To preserve the shape of the details we constrain  $T_i$  to **rotations, uniform scales and translations**

$$T_i = \begin{pmatrix} t_{11} & t_{12} & t_{13} & t_{14} \\ t_{21} & t_{22} & t_{23} & t_{24} \\ t_{31} & t_{32} & t_{33} & t_{34} \\ t_{41} & t_{42} & t_{43} & t_{44} \end{pmatrix}$$

Linear constraints on  $t_{lm}$   
so that  $T_i$  is  
rotation+scale+translation  
??

### Constraining $T_i$ – 3D case

- Not linear in 3D:

$$\begin{pmatrix} \text{rotation +} \\ \text{uniform scale} \end{pmatrix} = s \exp H = s(\alpha I + \beta H + \mathbf{h}^T \mathbf{h})$$

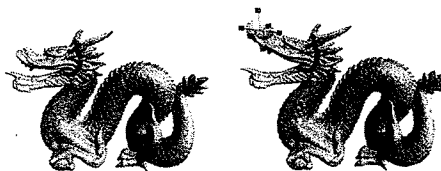
$H$  is  $3 \times 3$  skew-symmetric,  $H\mathbf{x} = \mathbf{h} \times \mathbf{x}$

- Linearize by dropping the quadratic term

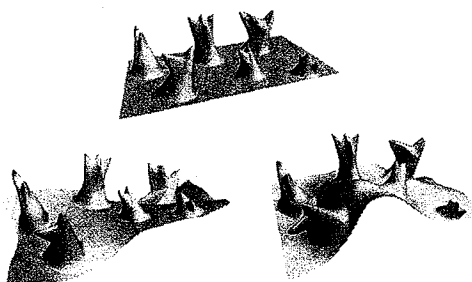
### Adjusting $T_i$

- Due to linearization,  $T_i$  scale the space along the  $\mathbf{h}$  axis by  $\cos\theta$
- When  $\theta$  is large, this causes anisotropy
- Possible correction:
  - Compute  $T_i$ , remove the scaling component and reconstruct the surface again from the corrected  $\delta_i$
  - Apply our technique from [Lipman et al. 04] first, and then the current technique – with small  $\theta$ .

### Some results



### Some results



### Some results

